Math 222 Keys and Hints for HW8

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Section 11.9

1, 6, 9, 13, 14, 18, 21, 22, 23, 28, 35, 36

1. Ans: Applying $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ here, we can get $e^{-5x} = \sum_{n=0}^{\infty} \frac{(-5x)^n}{n!}$

6. Ans: $\cos(x^{3/2}/\sqrt{2}) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^{3/2}/\sqrt{2})^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{2^n (2n)!}$

9. Ans:$\frac{x^2}{2} - 1 + \cos x = \frac{x^2}{2} - 1 + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=2}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

13. Ans:$\cos^2 x = \frac{1}{2}(1 + \cos 2x) = \frac{1}{2}(1 + \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{2(2n)!})$

14. Ans:Similar to above $\sin^2(x) = \frac{1}{2}(1 - \cos(2x)) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2x)^{2n}}{2(2n)!})$

18. Ans:$\frac{2}{(1-x)^3}$. You can calculate the derivatives directly. Here I'd like to do like this: $(\frac{1}{1-x})'' = (\sum_{n=0}^{\infty} x^n)'' = \sum_{n=2}^{\infty} n(n-1)x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1)x^n$

21. Ans:By the Taylor theorem, we use P_2 here, we have $\sin x = x + R_2 = x + \frac{-\cos(c)}{3!}x^3$. $|error| < \frac{10^{-9}}{6}$. If $x < \sin x$, we have $R_2 > 0$, which means $x < 0$.

22. Ans:We have the formula $\sqrt{1+x} = 1 + x/2 + \frac{f^{(2)}(c)}{2!}x^2$. We can calculate that $f''(x) = -\frac{1}{4}(1+x)^{-3/2}$, so we have $|error| = |\frac{1}{8(1+c)^{3/2}}x^2| < \frac{1}{8(1-0.01)^{3/2}}0.01^2$

23. Ans:The error term is $e^c x^3/3!$, so we have $|error| < e^{0.1}(0.1^3)/6$. If you would like to do $e < 3$, it's OK, but I don't think this is good enough.

28. Ans:We have the formula $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for $|x| \leq 1$. You can get this for $|x| < 1$ by integrating $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ and then extend to $x = \pm 1$ by the continuity. Anyway, we have $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$, which is a convergent alternating series. If we keep N terms, namely, we use $\sum_{n=0}^{N-1} \frac{(-1)^n}{2n+1}$, then we have $|error| < \frac{1}{2N+1}$ by the conclusion of alternating series. If we want to keep 2 good decimal places, which means in $0.a_1 a_2 \ldots$, a_1 and a_2 should be good. Here I'm not quite clear what *good* should mean. I understand it by using the approximation $0.1194 \approx 0.199$ and $0.1195 \approx 0.120$, namely more than or equal to 5 contributes 1 to the previous digit and no contribution otherwise. Here the third digit is not good. $\frac{1}{2N+1} < 5 * 10^{-4}$. I think you can use $< 10^{-3}$ here.

35. Ans:Not hard but needs a little calculation. I'd like to omit it here. I just give the forth term as an examople. We should decide the coefficient of x^5 (Attention: The term begins with x, and the coefficient of x^4 is 0, so the fourth term is x^5).

$1 * \frac{1}{5!} + \frac{1}{2!} * (-1)\frac{1}{3!} + \frac{1}{4!} * 1 = -\frac{1}{30}$. You can also take the imaginary part of $e^{(1+i)x}$.
36. Ans:Similar to the above. Omitted.

Section 9.1
1, 2, 6–8,9–14, 17, 19–22, 23, 24
1 and 2 can be done by differentiation. I'd like give 2 as an example: $y = -\frac{1}{x+c}$, so $y' = -(-1)\frac{1}{(x+c)^2}(x+c)' = y^2$.
6 to 8 can be done directly. I'd like to take 8 as an example. Check the initial value first: $y(e) = e/\ln e = e$, which is correct. Then the equation: $y' = \frac{1}{\ln x} - \frac{1}{(\ln x)^2}$ and you can see $x^2y' = xy - y^2$ is right.
9-14 and 17 are all separable equations.
9. Ans:$\sqrt{y}dy = \frac{1}{2\sqrt{x}}dx$. Then, integrate, and we have $\frac{2}{3}y^{3/2} = \sqrt{x} + C$
10. Ans:$2\sqrt{y} = \frac{1}{3}x^3 + C$
11. Ans:$e^y dy = e^x dx$, so we have $e^y = e^x + C$
12. Ans:$e^y dy = 3x^2 dx$. $\int e^y dy = \int 3x^2 dx$, which is $e^y = x^3 + C$
13. Ans:Firstly, we should separate the variables. We have $\frac{dy}{\sqrt{y}\cos^2\sqrt{y}} = dx$. Then integrate, and we have $\int \frac{1}{\sqrt{y}\cos^2\sqrt{y}}dy = \int dx$. On the left hand side, we do the substitution $u = \sqrt{y}$. We can have $2\arctan(\sqrt{y}) = x + C$
14. Ans:$\frac{2}{3}x^{3/2} = \sqrt{2x} + C$
17. Ans:$\arcsin(y) = x^2 + C$, which is $y = \sin(x^2 + C)$
19. d 20. c 21.a 22.b
23 and 24 are omitted.