Math 222 Keys and Hints for HW7

By Lei   October 21, 2010

Section 11.7
37, 38
37. Ans: Use the ratio test. We can see that the ratio is $\frac{x^2+1}{3}$. We first let the ratio be less than 1 to find the radius of convergence. $x^2 < 2$ and thus $-\sqrt{2} < x < \sqrt{2}$. However, if we set $x^2 = 2$, we can see that for both x's, the series diverges. Hence the interval is $(-\sqrt{2}, \sqrt{2})$. On this interval, the original series is a geometric series. The sum is $\frac{1}{1-(x^2+1)/3} = \frac{3}{2-x^2}$
38. Ans: the method is the same as 37. Interval $(-\sqrt{3}, \sqrt{3})$. The sum is $\frac{2}{3-x^2}$.

Section 11.8
7, 9, 10, 20, 21, 25, 26, 28
7. Ans: Find the different orders of derivatives of $f(x) = \sqrt{x}$, we have $f'(x) = \frac{1}{2}x^{-1/2}, f''(x) = \frac{-1}{4}x^{-3/2}, f^{(3)}(x) = \frac{3}{8}x^{-5/2}$. We have the polynomials of order 0,1,2,3 at 4 are $2, 2 + \frac{1}{4}(x-4), 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2, 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$
9. Ans:You can get the derivates to calculate. Here I just want to apply the conclusion for e^x. Replace x with $-x$, we have $\sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$
10. Ans: The method is similar with 9. We have the conclusion as $\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$
20. Ans: $x^2 + 2x + 1$
21. Ans:$f'(x) = 3x^2 - 2, f''(x) = 6x$ and $f'''(x) = 6$. The higher order derivatives are all 0. Hence the answer is $8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$
25. Ans:You can calculate the derivatives to calculate this. Here, I want to do something different. $f(x) = \frac{1}{x^2} = -(\frac{1}{1+(x-1)})' = -(\sum_{n=0}^{\infty}(-1)^n(x-1)^n)' = -(\sum_{n=0}^{\infty}(-1)^n n(x-1)^{n-1}) = \sum_{n=0}^{\infty}(-1)^{n+1}n(x-1)^{n-1} = \sum_{n=1}^{\infty}(-1)^{n+1}n(x-1)^{n-1} = \sum_{n=0}^{\infty}(-1)^{n+2}(n+1)(x-1)^n = \sum_{n=0}^{\infty}(-1)^n(n+1)(x-1)^n$
26. Ans:I also don't want to calculate the derivatives. $\frac{x}{1-x} = x\sum_{n=0}^{\infty} x^n = \sum_{n=1}^{\infty} x^n$
28. Ans:This is easy. We have two ways. The first is always to use the formula. Here I want to do something different. $2 * 2^{x-1} = 2 * e^{(x-1)\ln 2} = 2 * \sum_{n=0}^{\infty} \frac{(\ln 2)^n (x-1)^n}{n!} = \sum_{n=0}^{\infty} \frac{2(\ln 2)^n (x-1)^n}{n!}$