Math 222 Keys and Hints for HW6

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Section 11.5
2,3,6,11,12,17,20,21,39–42
2. Ans: $\sum_{n=1}^{\infty} n^2e^{-n}$ converges, because the ratio $a_{n+1}/a_n = \frac{(n+1)^2e^{-(n+1)}}{n^2e^{-n}} = \frac{(n+1)^2e^{-1}}{n^2} \to e^{-1} < 1$.
You can also use the limit test, because e^{pn} $(p > 0)$ is much quicker than the power.
$n^2e^{-n}/e^{-n/2} \to 0$ and $\sum e^{-n/2}$ converges.
3. Ans: Diverges. The ratio is $a_{n+1}/a_n = (n+1)e^{-1} \to \infty > 1$. Also you can check that the n-th term goes to infinity.
6. Ans:Diverges. The n-th term a_n goes to e^{-2}, which is not zero. The root test can't apply here, because it goes to 1.
11. Ans:Converges. Limit Comparison test. $\frac{\ln n/n^3}{1/n^2} \to 0$. The ratio test and root test don't work here, because the ration and the n-th root both go to 1.
12. Ans:Converges. The root test. $\sqrt[n]{\frac{(\ln n)^n}{n^n}} = \frac{\ln n}{n} \to 0$
17. Ans:Converges. The ratio test. $a_{n+1}/a_n = \frac{(n+2)(n+3)}{(n+1)(n+2)(n+1)} \to 0$. You can also justify by using the limit test by choosing appropriate series that can be obtained by using the different speeds of going to infinity I once said. For example, you can use $\sum \frac{1}{n^2}$ to compare.
20. Ans:Converges. Actually $a_n = \frac{n(n+1)2^n}{3^n}$ and the n-th root goes to 2/3.
21. Ans:Converges. For the term containing factorial, you can try ratio test. Here $a_{n+1}/a_n = \frac{n+1}{(2n+2)(2n+3)} \to 0$
39. Ans:Diverges.The term contains power, and you can try n-th root, which is $n!/n^2 \to \infty$.
40. Ans:Converges. The n-th root is $n!/n^n \to 0$
41. Ans:Converges. The n-th root is $n/2^n \to 0$
42. Ans:Diverges. The n-th root is $n/2^2 \to \infty$

Section 11.6
3,5,6,7,8,13,16,18,23,26,35,40,45,46,51
3. Ans:Diverges. The n-th term does NOT go to zero.
5. Ans:Converges.The Alternating Series Test.
6. Ans:Converges. The Alternating Series Test.
7. Ans:Diverges. $a_n = (-1)^{n+1}\frac{\ln n}{2\ln n} = (-1)^{n+1}\frac{1}{2}$ which doesn't go to zero.
8. Ans:Converges. Let $f(x) = \ln(1 + 1/x)$, and then $f'(x) = \frac{x}{x+1}\frac{-1}{x^2} < 0$. We conclude that the absolute value decreases. The limit is obviously 0. Using the Alternating Series Test.
13. Ans:Converges, but doesn't converge absolutely. Reason omitted here.
16. Ans:Diverges. Look at the n-th term.
18. Ans:Converges absolutely. The absolute value of the n-th term is less than or equal to $1/n^2$.

23. Ans:Converges absolutely. Look at the absolute value and use the root test.
26. Ans:Converges but doesn't converge absolutely. This is very easy, and find the reason yourself.
35. Ans:Converges absolutely. Easy, applying the n-th root to the absolute value.
40. Ans:Diverges. $a_n = (-1)^n \frac{n}{\sqrt{n^2+n}+n}$, and it doesn't go to 0.
45. Ans:The error $|s - s_4| < u_5 = 1/5$
46. Ans:$|s - s_4| < u_5 = 1/10^5 = 10^{-5}$
51. Ans:a. The absolute value u_n doesn't satisfy $u_n \geq u_{n+1}$ for large n.
b. This series converges absolutely and thus we can rearrage it's order and it is actually $\sum_{n=1}^{\infty} \frac{1}{3^n} - \sum_{n=1}^{\infty} \frac{1}{2^n} = 1/2 - 1 = -1/2$

Section 11.7

1,3,4,12,17,22,27,32

1. Ans:The ratio is x and thus it's limit is x. We require $ratio < 1$ and hence the radius of convergence is 1. For both $x = 1$ and $x = -1$, the series diverges. So for $-1 < x < 1$, it converges absolutely and otherwise it diverges.
3. Ans:The ratio is $(-1)(4x + 1)$. We require $|4x + 1| < 1$, which is $-1/2 < x < 0$ and hence the radius of convergence is 1/4. Again for -1/2 and 0,it diverges.
4. Ans:$1/3 < x < 1$ absolutely; 1/3, conditionally; others diverges. Radius: 1/3.
12. Ans:Ratio is $3x/(n + 1) \to 0$. Radius is infinity and for every x, the convergence is absolutely.
17. Ans: The limit of the ratio is $(x + 3)/5$. Radius 5. $-8 < x < 2$ absolutely; others diverges.
22. Ans:The limit of the ratio is x. Radius 1. $-1 < x < 1$ absolutely and others diverges.
27. Ans:Radius 1. For $-1 \leq x \leq 1$ converges absolutely. Others diverges.
32. Ans:Ratio $(x - \sqrt{2})^2/2$. We require $|x - \sqrt{2}| < \sqrt{2}$, hence the radius is $\sqrt{2}$. For $x = 2\sqrt{2}$, it diverges obviously and for 0, also diverges. Hence, finally we have $0 < x < 2\sqrt{2}$ for absolute convergence and others divergence.