Math 222 Keys and Hints for HW5

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Section 11.1
24. $a_n = \frac{n+(-1)^n}{n}$
Ans: Converges to 1.
26. $a_n = \frac{2n+1}{1-3\sqrt{n}}$
Ans: Diverges to negative infinity.
36. $a_n = (-\frac{1}{2})^n$
Ans: Converges to 0.
52. $a_n = \sqrt[n]{n^2}$
Ans: Converges to 1.
Hint: $\sqrt[n]{n} \to 1$ as $n \to \infty$
63. $a_n = (\frac{1}{n})^{1/\ln n}$
Ans: Converges to e^{-1}
Hint: $a_n = (\frac{1}{n})^{1/\ln n} = e^{\ln((\frac{1}{n})^{1/\ln n})} = e^{\frac{1}{\ln n}\ln(1/n)} = e^{-1}$
74. $a_n = n(1 - \cos\frac{1}{n})$
Ans: 0
Hint: L'Hospital. $\frac{1-\cos\frac{1}{n}}{1/n} \to \frac{\sin(1/n)}{1} \to 0$

Section 11.2
For 9-14, they are all geometric serieses. The method is the same.Pay attention to whether it begins at 0 or 1.
9. $\sum_{n=1}^{\infty} \frac{7}{4^n}$
Ans: 7/3.
Hint: When the absolute value of the ration is less than 1, you can use $a_1/(1-r)$
10. Ans:$\frac{5}{1-(-1/4)} = 4$
11. Ans:$\frac{5}{1-1/2} + \frac{1}{1-1/3} = 11.5$
14. Ans: $\frac{2}{1-2/5} = 10/3$
15. $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$
Ans: 1.
Hint: $\frac{4}{(4n-3)(4n+1)} = \frac{1}{4n-3} - \frac{1}{4n+1} = \frac{1}{4n-3} - \frac{1}{4(n+1)-3}$
18. Ans: $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \sum_{n=1}^{\infty}(\frac{1}{n^2} - \frac{1}{(n+1)^2}) = 1$
22. Ans: $\pi/4 - \pi/2 = -\pi/4$
24. Ans: Diverges because the ratio of the geometric series is bigger than 1.
28. Ans: Converges. It's actually $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n} = \frac{1}{1-(-1/5)} = 5/6$

29. Ans: Converges. It's also geometric. $\frac{1}{1-e^{-2}}$

37. Ans:Diverges. $\sum_{n=1}^{\infty}(\ln n - \ln(n+1)) = \lim_{n\to\infty}(-\ln(n+1))$

39. Ans: Converges. Geometric and $e/\pi < 1$. $\frac{1}{1-e/\pi}$

43. Ans: We require $|x-1| < 2$, namely $-1 < x < 3$. The sum is $\frac{3}{1-\frac{x-1}{2}} = \frac{6}{3-x}$. Here if $x = 1$, we define the zeroth term to be 3.

51. Ans: 23/99

Hint: $23/10^2 + 23/10^4 + 23/10^6 + \ldots$, a geometric series again.

56. Ans:$1 + \frac{414}{10^3} + \frac{414}{10^6} + \ldots = 1 + 414 * \frac{10^{-3}}{1-10^{-3}} = 1 + \frac{414}{999} = \frac{1413}{999}$

77. Ans:

In the first graph, we have 3 edges, and from the first graph to the second, each edge generates 4 edges, so we have 3*4 edges in the second graph. From the second to the third, each edge generates 4 edges, so we have 3*4*4 edges in the third. Generally, we have $3 * 4^{n-1}$ edges in the n-th graph.

The length of the edge in the first graph is 1, and the length of one edge in the second is 1/3. In the third, it's $(1/3)^2$ and so on. Generally, the length of one edge in the n-th graph is $(\frac{1}{3})^{n-1}$.

We have $L_n = 3 * 4^{n-1} * (\frac{1}{3})^{n-1} = 3 * (\frac{4}{3})^{n-1} \to \infty$.

Let's see the areas. Suppose $B_n = A_{n+1} - A_n$, and we have B_1 as the sum of the areas of the three triangles with length of 1/3, so $B_2 = 3 * (1/3)^2 * A_1$. From n to n+1, each edge genarates one small triangle. We have $3 * 4^{n-1}$ edges in the n-th graph, and the length of the edge in the (n+1)th graph is $(\frac{1}{3})^n$. Thus we have the area increasement from the nth to the (n+1)th as $3 * 4^{n-1} * [(\frac{1}{3})^n]^2 * A_1 = \frac{1}{3} * (4/9)^{n-1} * A_1$, so we have:

$A_n = A_1 + B_1 + B_2 + B_3 + \ldots + B_{n-1}$ and

$A_n \to A_1 + \frac{1}{3}A_1 + \frac{1}{3}(4/9)A_1 + \ldots = A_1 + \frac{A_1}{3}\sum_{n=1}^{\infty}(\frac{4}{9})^{n-1} = A_1 + \frac{A_1}{3}\frac{1}{1-4/9} = A_1 + A_1 * \frac{3}{5}$.

However, $A_1 = \frac{1}{2} * 1 * 1 * \sin(\pi/3) = \sqrt{3}/4$. We have the limit as $2\sqrt{3}/5$.

Section 11.3

3. Diverges because $a_n = n/(n+1) \to 1$ rather than 0.

6. Converges because it's equal to $-2 * (p - series)$ where $p = 3/2$.

16. Diverges. You can use integral test, but I think check the order of the n is best. The order is 1, so it acts like $1/n$. You can use the limit comparison test to justify this,

20. Converges. $\ln 3 > 1$ and it's a geometric series. $1/\ln 3 < 1$.

22. Converges. Integral test. $\int_1^{\infty}\frac{1}{x(1+(\ln x)^2)}\mathrm{d}x$ converges.

25. Converges. Integral test.

39. it is $\lim_{b\to\infty}\frac{1}{1-p}(\ln x)^{1-p}|_2^b$ $(p \neq 1)$ or $\lim_{b\to\infty}\ln(\ln x)|_2^b$ (p=1) converges if and only if $p > 1$. The series converges if and only if $p > 1$.

Section 11.4

2. Diverges. It acts like $1/n$, because $\lim_{n\to\infty}\frac{n+\sqrt{n}}{n} = 1$. Limit Comparison test.

7. Converges, because $\frac{n}{3n+1} < \frac{1}{3}$

Qs:How about $\sum_{n=1}^{infty}(\frac{3n}{3n+1})^n$?(Diverges)
10. Diverges. $(\ln n)^p/n^q \to 0$ for any positive numbers p and q.
11. Converges. The reason is the same as #10.
19. Converges. The order is 2. Use the limit Comparison Test.
20. Converges. The order is 3/2. Thus you can use $\lim_{n\to\infty}(\frac{\sqrt{n}}{n^2+1})/(\frac{1}{n^{3/2}})$
21. Converges. You can move one minus sign out and then you can see that it will goes like $1/2^n$. Use the limit comparison test to justify this.
28. Converges. The order of the n is $2+1+2-3=2$, so you can check the limit $a_n/(1/n^2)$
34. Converges. Limit Comparison Test.