Keys to HW 3.

By Lei. Sep. 27

(11) $\int \frac{\sqrt{y^2-49}}{y} dy \quad (y>7)$

Ans: $\sqrt{y^2-49} - 7\sec^{-1}\left(\frac{y}{7}\right) + C$

Hint: $y = 7\sec\theta$. Choose $0 < \theta < \frac{\pi}{2}$

we can have $\tan\theta > 0$.

So we have $dy = 7\tan\theta\sec\theta \cdot d\theta$.

$$\int \frac{7\tan\theta}{7\sec\theta}(7\tan\theta\sec\theta)\cdot d\theta$$

$$= \int 7\tan^2\theta \cdot d\theta = 7\int(\sec^2\theta - 1)\cdot d\theta$$

(12) $\int \frac{\sqrt{y^2-25}}{y^3} dy \quad (y>5)$

Ans: $\frac{1}{10}\sec^{-1}\left(\frac{y}{5}\right) - \frac{\sqrt{y^2-25}}{2y^2} + C$

Hint: $y = 5\sec\theta$.

$$\int \frac{5\tan\theta}{5^3\sec^3\theta}(5\tan\theta\sec\theta)\cdot d\theta$$

$$= \frac{1}{5}\int \sin^2\theta \cdot d\theta = \frac{1}{5}\int \frac{1}{2}[1-\cos 2\theta]\cdot d\theta$$

(14). $\int \frac{2\,dx}{x^3\sqrt{x^2-1}} \quad x>1.$

Ans: $\sec^{-1}(x) + \frac{\sqrt{x^2-1}}{x^2} + C$

Hint: $x = \sec\theta > 1$.

$$\int \frac{2\tan\theta\sec\theta\cdot d\theta}{\sec^3\theta\cdot\tan\theta} = \int 2\cos^2\theta\cdot d\theta$$

$$= \theta + \frac{1}{2}\sin 2\theta + C$$

(26) $\int \frac{6\,dt}{(9t^2+1)^2}$

Ans: $\frac{3t}{9t^2+1} + \tan^{-1}(3t) + C$

Hint: $3t = \tan\theta$.

$$\int \frac{6\cdot(\frac{1}{3}\sec^2\theta)\cdot d\theta}{\sec^4\theta} = \int 2\cos^2\theta\cdot d\theta$$

$$= \frac{1}{2}\sin 2\theta + \theta + C$$

$$= \frac{1}{2}\cdot\frac{2\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta} + \theta + C = \frac{\tan\theta}{\tan^2\theta+1} + \theta + C$$

Sec. 8.8.

(1) $\int_0^\infty \frac{dx}{x^2+1}$

Ans: $\pi/2$

(2) $\int_1^\infty \frac{dx}{x^{1.001}}$

Ans: 1000.

(3) $\int_0^1 \frac{dx}{\sqrt{x}}$.

Ans: 2.

(4) $\int_0^4 \frac{dx}{\sqrt{4-x}}$.

Ans: 4.

Hint: at $x=4$, $\frac{1}{\sqrt{4-x}}$ is not continuous.

So $\lim_{b\to 4^-}\int_0^b \frac{1}{\sqrt{4-x}}dx = \lim_{b\to 4^-}\left(-2\sqrt{4-x}\right)\Big|_0^b$

$= \lim_{b\to 4^-}\left(-2\sqrt{4-b} + 4\right) = 4.$

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$$= \lim_{b\to 4^-}\left(-2\sqrt{4-b} + 4\right) = 4.$$

(6) $\int_{-8}^{1} \frac{dx}{x^{1/3}}$

Ans: $-\frac{9}{2}$

Hint: $\frac{1}{x^{1/3}}$ is not continuous at $x=0$. So the integral is

$$\lim_{a\to 0^-}\int_{-8}^{a} \frac{1}{x^{1/3}}dx + \lim_{b\to 0^+}\int_{b}^{1} \frac{1}{x^{1/3}}dx$$

$$= \lim_{a\to 0^-}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_{-8}^{b}\right) + \lim_{b\to 0^+}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_{b}^{1}\right)$$

$$= \lim_{a\to 0^-}\left[\frac{3}{2}a^{\frac{2}{3}} - \frac{3}{2}\sqrt[3]{(-8)^2}\right] + \left[\frac{3}{2} - \lim_{b\to 0^+}\frac{3}{2}b^{\frac{2}{3}}\right]$$

$$= -\frac{3}{2}\times 4 + \frac{3}{2}$$

(7). $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Ans: $\pi/2$.

(24). $\int_{-\infty}^{+\infty} 2xe^{-x^2}dx$

Ans: 0

Hint: $\int_{-\infty}^{0} + \int_{0}^{+\infty}$.

Another way is by noticing that $2xe^{-x^2}$ is odd on $(-\infty, +\infty)$.

(25). $\int_0^1 x\ln x \cdot dx$

Ans: $-1/4$.

(35) $\int_0^{\pi/2} \tan\theta \cdot d\theta$

Ans: Diverges.

Hint: Integration.

$$\lim_{b\to\pi/2}\left(\ln|\sec\theta|\Big|_0^b\right)$$

$$= \lim_{b\to\pi/2}\left(\ln|\sec b|\right) = +\infty$$

(40). $\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$

Ans: Converges

(53). $\int_1^{\infty} \frac{\sqrt{x+1}}{x^2} dx$.

Ans: Converges.

Hint: Since $\lim_{x\to+\infty} \frac{\left(\frac{\sqrt{x+1}}{x^2}\right)}{x^{3/2}}$

$$= \lim_{x\to+\infty}\frac{\sqrt{x+1}}{\sqrt{x}} = 1$$, and

$\int_1^{\infty} \frac{1}{x^{3/2}}dx$ converges.

(56). $\int_{\pi}^{\infty} \frac{1+\sin x}{x^2}dx$

Ans: converges.

Hint: $0 \le \frac{1+\sin x}{x^2} \le \frac{2}{x^2}$

(58). $\int_2^{\infty} \frac{1}{\ln x}dx$.

Ans: Diverges.

Hint: When $x \geq 2$. $\frac{1}{x} \le \frac{1}{\ln x}$

Sec. 11.1

1, 2, 4. omitted.

13. 1, −1, 1, −1, 1. ⋯

Ans: $a_n = (-1)^{n-1}$

16. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \cdots$

Ans: $a_n = \frac{(-1)^{n-1}}{n^2}$

20: 2, 6, 10, 14, 18, ⋯

Ans: $a_n = 4n - 2$.

Hint: By noticing that

$a_{n+1} - a_n = 4$, we have

$a_n = 2 + 4 \times (n-1)$

31. $a_n = 1 + (-1)^n$

Ans: Diverges.

Hint: 0, 2, 0, 2, 0, 2, ⋯

For any number L and any integer N, all $n \geq N$, $|a_n - L|$ can't be arbitrarily small.

41. $a_n = \frac{\sin n}{n}$

Ans: Converges to 0.

Hint: $-\frac{1}{n} \leq \frac{\sin n}{n} \leq \frac{1}{n}$.

Sandwich.