Math 222 Keys and Hints for HW3

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Section 8.3

21. $\int_0^1 \frac{\mathrm{d}x}{(x+1)(x^2+1)}$

Ans: $(\pi + 2\ln 2)/8$

Hint: $\frac{\mathrm{d}x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$, so we have $A(x^2+1) + (Bx+C)(x+1) = 1$. Let $x \to -1$, we have $A = 1/2$. Differentiate, we can further have $B = -1/2, C = 1/2$. The antidirevative is $0.5\ln|x+1| - 0.25\ln|x^2+1| + 0.5\tan^{-1} x + C$

22. $\int_1^{\sqrt{3}} \frac{3t^2+t+4}{t^3+t}\mathrm{d}t$

Ans: $2\ln 3 - \frac{1}{2}\ln 2 + \pi/12$

Hint: $t(t^2+1)$, since t^2+1 is irreducible, we can have $\frac{3t^2+t+4}{t^3+t} = \frac{A}{t} + \frac{Bt+C}{t^2+1}$. Thus we have $3t^2+t+4 = A(t^2+1) + t(Bt+C)$. $t \to 0$ implies $A = 4$. $4 + B = 3$, $B = -1$. $C = 1$

23. $\int \frac{y^2+2y+1}{(y^2+1)^2}\mathrm{d}y$

Ans: $\tan^{-1} y - \frac{1}{y^2+1} + C$

Hint: This problem can be solved directly by observing that $y^2 + 2y + 1 = (y^2+1) + 2y$

26. $\int \frac{s^4+81}{s(s^2+9)^2}\mathrm{d}s$

Ans: $\ln|s| + \frac{9}{s^2+9} + C$

Hint: $\frac{s^4+81}{s(s^2+9)^2} = \frac{A}{s} + \frac{Bs+C}{(s^2+9)^2} + \frac{Ds+E}{s^2+9}$. So we have $s^4 + 81 = A(s^2+9)^2 + (Bs+C)s + (Ds+E)s(s^2+9)$. $s \to 0$, we have $A = 1$. Then $0 = 18s + Bs + C + (Ds+E)(s^2+9)$. Because there are no cubed and squared on the left, we must have $D = E = 0$ and thus $B = -18, C = 0$.

28. $\int \frac{\theta^4-4\theta^3+2\theta^2-3\theta+1}{(\theta^2+1)^3}\mathrm{d}\theta$

Ans: $\arctan\theta + \frac{2}{\theta^2+1} - \frac{1}{4(\theta^2+1)^2} + C$

Hint: Trying to get θ^2+1 in the numerator, we can observe that it is $(\theta^2+1)^2 - 4\theta(\theta^2+1) + \theta$

33. $\int \frac{y^4+y^2-1}{y^3+y}\mathrm{d}y$

Ans: $y^2/2 - \ln|y| + \frac{1}{2}\ln(y^2+1) + C$

Hint: Check the degree and we can see that it's an improper fraction. We have it as $y - \frac{1}{y(y^2+1)} = y - \frac{(y^2+1)-y^2}{y(y^2+1)}$

36. $\int \frac{e^{4t}+2e^{2t}-e^t}{e^{2t}+1}\mathrm{d}t$

Ans: $e^{2t}/2 + \frac{1}{2}\ln|e^{2t}+1| - \arctan(e^t) + C$

Hint: Substitution $u = e^t$

46. The volume of the solid generated by revolving the region in $y = \frac{2}{(x+1)(2-x)}$, $x = 1$ and $x = 0$ about the y-axis($x = 0$).

Ans: $4\pi\ln 2/3$

Hint: $\int_0^1 2\pi x y(x)\mathrm{d}x$

Section 8.4
3. $\int_{-\pi/2}^{\pi/2} \cos^3 x \mathrm{d}x$
Ans: 4/3
Hint: $(1-\sin^2 x)\cos x$ and then substitute $u = \sin x$
7. $\int_0^{\pi} 8\sin^4 x \mathrm{d}x$
Ans: 3π
Hint: $\sin^2 x = (1-\cos(2x))/2$ and we have $2 - 4\cos 2x + 2\cos^2(2x)$. The first is 2π and the second must be zero. The last is the integral of $1-\cos(4x)$.
10. $\int_0^{\pi} 8\sin^4 y \cos^2 y \mathrm{d}y$
Ans: $\pi/2$
Hint: All degrees are even. It is
$(1-\cos(2y))^2(1+\cos(2y)) = (1-\cos(2y))(\sin^2(2y)) = \sin^2(2y) - \sin^2(2y)\cos(2y)$. The first is half of length, which is $\pi/2$ and the second can be done by $u = \sin(2y)$, which is zero.
12. $\int_0^{\pi} \sin(2x)\cos^2 2x \mathrm{d}x$
Ans: 0
Hint: $u = \cos 2x$ or notice that the center is $\pi/2$ and on each side, the function differs by a sign
16. $\int_0^{\pi} \sqrt{1-\cos 2x}\mathrm{d}x$
Ans: $2\sqrt{2}$
Hint: $1-\cos 2x = 2\sin^2 x$ and $\sin x$ is positive on the interval.
20. $\int_{-\pi/4}^{\pi/4} \sqrt{\sec^2 x - 1}\mathrm{d}x$
Ans: $\ln 2$
Hint: $\sec^2 x - 1 = \tan^2 x$, so $\int_{-\pi/4}^{\pi/4} |\tan x|\mathrm{d}x = 2\int_0^{\pi/4} \tan x \mathrm{d}x$
23. $\int_{-\pi/3}^{0} 2\sec^3 x \mathrm{d}x$
Ans: $2\sqrt{3} + \ln(2+\sqrt{3})$
Hint: $\sec x$ is even, so the integral is equal to $I = \int_0^{\pi/3} 2\sec^3 x \mathrm{d}x$.
However, by integral by parts, $I = \int_0^{\pi/3} 2\sec x \mathrm{d}\tan x =$
$(2\sec x \tan x)|_0^{\pi/3} - \int_0^{\pi/3} 2\sec^x(\tan^2 x)\mathrm{d}x = (2\sec x\tan x)|_0^{\pi/3} - I + \int_0^{\pi/3} 2\sec^x \mathrm{d}x$
Here I is a number, so we have $I = (\sec x \tan x)|_0^{\pi/3} + \ln|\sec x + \tan x||_0^{\pi/3}$.
25. $\int_0^{\pi/4} \sec^4 \theta \mathrm{d}\theta$
Ans: 4/3
Hint: $(\tan^2 x + 1)\sec^2 x$ and $u = \tan x$
30. $\int_{-\pi/4}^{\pi/4} 6\tan^4 x \mathrm{d}x$
Ans: $3\pi - 8$
Hint: $6\tan^2 x(\sec^2 x - 1)$

Section 8.5
1. $\int \frac{1}{\sqrt{9+y^2}}\mathrm{d}y$
Ans: $\ln|\sqrt{9+y^2} + y| + C$

Hint: One way is to use the formula in Sec. 8.1 and the other one is $u = 3\tan\theta$
7. $\int \sqrt{25-t^2}\mathrm{d}t$
Ans: $\frac{25}{2}\sin^{-1}(\frac{t}{5}) + \frac{t\sqrt{25-t^2}}{2} + C$
Hint: $t = 5\sin\theta$
8. $\int \sqrt{1-9t^2}\mathrm{d}t$
Ans: Omitted.
Hint: $3t = \sin\theta$. Since the method is similar. I'd like to omit the answer here.
15. $\int \frac{x^3}{\sqrt{x^2+4}}\mathrm{d}x$
Ans: $\frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$
Hint: $x = 2\tan\theta$
16. $\int \frac{1}{x^2\sqrt{x^2+1}}\mathrm{d}x$
Ans: Omitted.
Hint: $x = \tan\theta$
18. $\int \frac{\sqrt{9-w^2}}{w^2}\mathrm{d}w$
Ans: Omitted.
Hint: $w = 3\cos\theta$
28. $\int \frac{(1-r^2)^{5/2}}{r^8}\mathrm{d}r$
Ans: $-\frac{(1-r^2)^{7/2}}{7r^7} + C$
Hint: $r = \cos\theta$ and it is $-\frac{\tan^7\theta}{7} + C$
32. $\int_1^e \frac{1}{y\sqrt{1+(\ln y)^2}}\mathrm{d}y$
Ans: $\ln|\sqrt{2}+1|$
Hint: $u = \ln y$ and we have $\int_0^1 \frac{1}{\sqrt{1+u^2}}\mathrm{d}u$