Math 222 Keys and Hints for HW2

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Section 8.2
$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$
4. $\int x^2 \sin x \mathrm{d}x$
Ans: $-x^2\cos x + 2x\sin x + 2\cos x + C$
Hint: Integral by part. Noticing that the derivatives of polynomials have lower degrees and we can calculate the integral of sin or cos easily, for polynomials multiplying sin or cos, we can use u being the polynomials and the remaining part being dv.
Here, we first let $u = x^2, dv = \sin x dx$ and then let $u = x$ and $dv = \cos x dx$.
6. $\int_1^e x^3 \ln x \mathrm{d}x$
Ans: $\frac{3e^4}{16} + \frac{1}{16}$
Hint: Letting $u = \ln x$ and $dv = x^3 dx$, we have $\int_1^e x^3 \ln x \mathrm{d}x = \frac{1}{4}x^4 \ln x|_1^e - \int_1^e \frac{1}{4}x^3 \mathrm{d}x$
13. $\int (x^2 - 5x)e^x \mathrm{d}x$
Ans: $(x^2 - 7x + 7)e^x + C$
Hint: $u = polynomial$ and $e^x dx = dv$.
22. $\int e^{-y} \cos y \mathrm{d}y$
Ans: $\frac{1}{2}(\sin y e^{-y} - \cos y e^{-y}) + C$
Hint: Either letting $u = \cos y, dv = e^{-y}dy$ or letting $u = e^{-y}, dv = \cos y dy$ can help you. However, you must do in one direction. You'll have
$\int e^{-y}\cos y \mathrm{d}y = \sin y e^{-y} - \cos y e^{-y} - \int e^{-y} \cos y \mathrm{d}y$. Don't forget the constant term when move the integral to the left, why?
28. $\int \ln(x + x^2)\mathrm{d}x$
Ans: $x\ln(x + x^2) + \ln|x + 1| - 2x + C$
Hint: $u = \ln(x + x^2), dv = dx$, we have $\int \ln(x + x^2)\mathrm{d}x = x\ln(x + x^2) - \int (2 - \frac{1}{1+x})\mathrm{d}x$.
29. $\int \sin(\ln x)\mathrm{d}x$
Ans: $\frac{1}{2}[-x\cos(\ln x) + x\sin(\ln x)] + C$
Hint: Substitution $u = \ln x$, we have $du = \frac{1}{x}dx$ and $dx = xdu = e^u du$. Thus
$\int \sin(\ln x)\mathrm{d}x = \int \sin(u)e^u du\mathrm{d}u$. Then, integrate by part.
30. $\int z(\ln z)^2 \mathrm{d}z$
Ans: $\frac{1}{2}z^2(\ln z)^2 - \frac{1}{2}z^2 \ln z + \frac{1}{4}z^2 + C$
Hint: $u = \ln z$, we have $z = e^u$ and $dz = e^u du$, so the integral is like this:
$\int z(\ln z)^2 \mathrm{d}z = \int e^u u^2 e^u \mathrm{d}u = \int e^{2u}u^2 \mathrm{d}u$
33.Volume of the solid generated by revolving the region bounded by $y = e^x$, axes and $x = \ln 2$ about $y = \ln 2$.
Ans: $2\pi(1 - \ln 2)$
Hint: I don't know how to draw the figure using Latex, so I won't draw the figure here. One way is by using the formula: $\int_{lowerlimit}^{upperlimit} \pi r^2(s)\mathrm{d}s$ along the rotation axis. By this way, we have $\int_0^1 \pi(\ln 2)^2 \mathrm{d}y + \int_1^2 \pi(\ln 2 - \ln y)^2 \mathrm{d}y$.

Another good way, which is like this (One student(Seemed Zain) reminded me of this): Dividing the solid into many layers and the volume of each one is $2\pi(\ln 2 - x) * e^x dx$, so the volume is $\int_0^{\ln 2} 2\pi(\ln 2 - x)e^x \mathrm{d}x = \int_0^{\ln 2} 2\pi \ln 2 e^x \mathrm{d}x - \int_0^{\ln 2} 2\pi x e^x \mathrm{d}x = 2\pi(1 - \ln 2)$.
34.(a) Volume of the solid generated by revolving the region bounded by $y = e^{-x}$, axes and $x = 1$ about y-axis($x = 0$).
Ans: $2\pi(1 - 2e^{-1})$
Hint: The method is similar to #33.
44. $\int \tan^{-1} x \mathrm{d}x$
Ans: $x \tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + C$
Hint: $u = \tan^{-1} x$ and $dv = dx$.

Section 8.3
1. $\frac{5x-13}{(x-3)(x-2)}$
Ans: $\frac{2}{x-3} + \frac{3}{x-2}$
Hint: Cover-up method.
4. $\frac{2x+2}{x^2-2x+1}$
Ans: $\frac{4}{(x-1)^2} + \frac{2}{x-1}$
Hint: You can use the general conclusion, but here we want to do like this:$2x + 2 = 2x - 2 + 4$.
6. $\frac{z}{z^3-z^2-6z}$
Ans: $\frac{1}{5}(\frac{1}{z-3} - \frac{1}{z+2})$
Hint: Cancel z in the numerator and denominator, and then use cover-up.
13. $\int_4^8 \frac{y}{y^2-2y-3}\mathrm{d}y$
Ans: $\frac{\ln 15}{2}$
Hint: $\frac{3/4}{y-3} + \frac{1/4}{y+1}$ and we have $\frac{1}{4}\ln[(y-3)^3(y+1)]|_4^8$.
16. $\int \frac{x+3}{2x^3-8x}\mathrm{d}x$
Ans: $-\frac{3}{8}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{5}{16}\ln|x-2| + C$
Hint: $\frac{x+3}{2x^3-8x} = \frac{1}{2}(\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2})$ and then $A = -3/4, B = 1/8, C = 5/8$ by cover-up.
19. $\int \frac{1}{(x^2-1)^2}\mathrm{d}x$
Ans: $\frac{1}{4}\ln|\frac{x+1}{x-1}| - \frac{x}{2(x^2-1)} + C$
Hint: Write out the form: $\frac{1}{(x^2-1)^2} = \frac{A}{(x-1)^2} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{x+1}$, and then mutiplying $(x-1)^2$, letting $x \to 1$, we have $A = 1/4$. Simiarly, we have $B = 1/4$. Subtracting these two terms, we have $\frac{1}{2(1+x)(1-x)} = \frac{C}{x-1} + \frac{D}{x+1}$, so $-C = D = 1/4$.
20. $\int \frac{x^2}{(x-1)(x^2+2x+1)}\mathrm{d}x$
Ans: $\frac{1}{4}\ln|x-1| + \frac{1}{2(x+1)} + \frac{3}{4}\ln|x+1| + C$
Hint: $\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{1/4}{x-1} + \frac{-1/2}{(x+1)^2} + \frac{3/4}{x+1}$.

29. $\int \frac{2x^3-2x^2+1}{x^2-x}\mathrm{d}x$
Ans: $x^2+\ln|\frac{x-1}{x}|+C$
Hint: The degree of the numerator isn't smaller that that of the denominator, so we shuld reduce this improper fraction first. By long division:
$\frac{2x^3-2x^2+1}{x^2-x}=2x+\frac{1}{x(x-1)}$
30. $\int \frac{x^4}{x^2-1}\mathrm{d}x$
Ans: $\frac{x^3}{3}+x+\frac{1}{2}\ln|\frac{x-1}{x+1}|+C$
Hint: Reducing the improper fraction, we have $\frac{x^4}{x^2-1}=x^2+1+\frac{1}{x^2-1}$.