Math 222 Keys and Hints for HW15

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I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 13.3

3,5,6,9,10,12,15,16

3, 5 and 6 are similar. I'll do 5 and 6.

5.Ans: $v = r' = -3\sin t\cos^2 tj + 3\sin^2 t\cos tk$. $|v| = \frac{3}{2}|\sin(2t)|$ and thus the unit tangent vector is $T = -\frac{|\cos t|\sin t}{|\sin t|}j + \frac{|\sin t|\cos t}{|\cos t|}k$. If $0 \le t \le \pi/2$, we have $T = -\cos tj + \sin t$. The length is $\int_0^{\pi/2}\frac{3}{2}\sin(2\tau)d\tau = 3/2$

6.Ans: $v =< 18t^2, -6t^2, -9t^2 >$. $|v| = 21t^2$. $T =< 6/7, -2/7, -3/7 >$. Length $\int_1^2 21\tau^2 d\tau = 49$

9.Ans:$|v(t)| = 13$. $(0, -12, 0)$ corresponds to $t = 0$. The direction of increasing the length means $t_2 > 0$ and thus $\int_0^{t_2} 13d\tau = 26\pi$, so $t_2 = 2\pi$. The point is $(0, 5, 24\pi)$

10. Ans:$|v| = 13$. We know the point corresponds to $t_1 = -\pi$. The point is $(0, 12, -5\pi)$

12.Ans: $|v| = t$ and thus $s = t^2/2$. The length is $s(\pi) - s(\pi/2) = 3\pi^2/8$

15. $t = 0 \to t = 1$, easy, omitted.

16. I can't draw the picture here, and thus this problem is omitted.

Section 13.4

5,9–12,17,20,21

5.a. Here, $v =< 1, f'(x) >$. We have $T = \frac{<1,f'(x)>}{\sqrt{1+(f'(x))^2}}$.

$\kappa = \frac{1}{|v|}|\frac{dT}{dx}| = \frac{1}{\sqrt{1+(f'(x))^2}}|\frac{<0,f''(x)>}{\sqrt{1+(f'(x))^2}} - \frac{1}{2}\frac{<1,f'(x)>}{\sqrt{1+(f'(x))^2}^3}2f'(x)f''(x)| =$
$\frac{1}{\sqrt{1+(f'(x))^2}}|\frac{<-f'(x)f''(x),f''(x)>}{\sqrt{1+(f'(x))^2}^3}| = \frac{|f''(x)|}{\sqrt{1+(f'(x))^2}^3}$

b and c are omitted, because it's quite simple. Note, at the point of inflection, $f''(x) = 0$

9-12 are similar. Use the formulae to calculate. I don't want to explain here.

20. a. $v =< -3\sin t, 3\cos t, 1 >$ and we have $|v| = \sqrt{10}$. $T =< -3\sin t, 3\cos t, 1 > /\sqrt{10}$. The curvature is 3/10. The total curvature is $\int_0^{4\pi} 0.3 * \sqrt{10}d\tau = 6\sqrt{10}\pi/5$

b. We parametrize the curve as $r = xi + x^2j$. Then $v =< 1, 2x >$ and $|v| = \sqrt{1 + 4x^2}$. $f''(x) = 2$ and $f'(x) = 2x$. Using the conclusion in 5, the curvature is $\kappa = \frac{2}{[1+4x^2]^{3/2}}$. We have $\int_{-\infty}^{\infty}\frac{2}{1+4x^2}dx = \pi$

21. Ans: The center is $(\pi/2, 1) + \frac{1}{\kappa}N$. $T =< 1, -\sin t > /\sqrt{1 + \cos^2 t}$ At $t = \pi/2$, $\kappa = 1$ $N =< 0, -1 >$. Center is $(\pi/2, 0)$. The circle is $(x - \pi/2)^2 + y^2 = 1$