I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 13.3
3, 5, 6, 9, 10, 12, 15, 16
3 and 6 are similar. I’ll do 5.

5. The center is \( \left( \frac{\pi}{2}, \frac{3}{2} \right) \sin(2t) \) and thus the unit tangent vector is \( T = -\frac{1}{\sqrt{1 + f'(t)^2}} \left| f''(t) \right| f''(t) \left( x, f'(x) \right) \). If \( 0 \leq t \leq \pi/2 \), we have \( T = -\cos t j + \sin t \). The length is \( \int_0^{\pi/2} \frac{3}{2} \sin(2\tau) d\tau = 3/2 \)

6. \( v = <18t^2, -6t^2, -9t^2> \). \( |v| = 21t^2 \). \( T = <6/7, -2/7, -3/7> \). Length \( \int_0^1 21t^2 dt = 49 \)

9. \( \int_0^1 |v(t)| = 13 \). \( 0, -12, 0 \) corresponds to \( t = 0 \). The direction of increasing the length means \( t_2 > 0 \) and thus \( \int_0^{t_2} 13 d\tau = 26\pi \), so \( t_2 = 2\pi \). The point is \((0, 0, 24\pi)\).

10. \( \int_0^1 |v| = 13 \). We know the point corresponds to \( t_1 = -\pi \). The point is \((0, 12, -5\pi)\).

12. \( \int_0^1 |v| = t \) and thus \( s = t^2/2 \). The length is \( s(\pi) - s(\pi/2) = 3\pi^2/8 \)

15. \( t = 0 \rightarrow t = 1 \), easy, omitted.

16. I can’t draw the picture here, and thus this problem is omitted.

Section 13.4
5, 9, 12, 17, 20, 21
5a. Here, \( v = <1, f'(x)> \). We have \( T = \frac{<1, f'(x)>}{\sqrt{1 + f'(x)^2}} \)

\[ \kappa = \frac{1}{|v|} \left| \frac{dT}{dx} \right| = \frac{1}{\sqrt{1 + f'(x)^2}} \left| \frac{<0, f''(x)>}{\sqrt{1 + f'(x)^2}} - \frac{1}{2} \left| \frac{<1, f'(x)>}{\sqrt{1 + f'(x)^2}} \right|^2 f''(x) f''(x) \left| f''(x) \right| = \]

\[ \frac{1}{\sqrt{1 + f'(x)^2}} \left| \frac{-<f'(x), f''(x)>}{\sqrt{1 + f'(x)^2}} \right|^2 = \frac{1}{\sqrt{1 + f'(x)^2}} \]

b and c are omitted, because it’s quite simple. Note, at the point of inflection, \( f''(x) = 0 \) and \( 9-12 \) are similar. Use the formulae to calculate. I don’t want to explain here.

20. \( v = <3\sin t, 3\cos t, 1> \) and we have \( |v| = \sqrt{10} \). \( T = <3\sin t, 3\cos t, 1> / \sqrt{10} \).

The curvature is \( 3/10 \). The total curvature is \( \int_0^{4\pi} 0.3 * \sqrt{10} d\tau = 6\sqrt{10}\pi/5 \)

b. We parametrize the curve as \( r = xi + x^2 j \). Then \( v = <1, 2x> \) and \( |v| = \sqrt{1 + 4x^2} \).

\( f''(x) = 2 \) and \( f'(x) = 2x \). Using the conclusion in 5, the curvature is \( \kappa = \frac{2}{\sqrt{1 + 4x^2}} \sqrt{1 + 4x^2} \).

We have \( \int_{-\infty}^{+\infty} \frac{1}{1 + 4x^2} dx = \pi \)

21. \( N = \left( \frac{\pi}{2}, 1 \right) + \frac{1}{\kappa} \). \( T = <1, -\sin t> / \sqrt{1 + \cos^2 t} \) At \( t = \pi/2, \kappa = 1 \)

\( N = <0, -1> \). Center is \((\pi/2, 0)\). The circle is \((x - \pi/2)^2 + y^2 = 1 \)