Math 222 Keys and Hints for HW14

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I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 12.5

17,20,27,28,39,42,45,46,61,62,70

17 and 20 are similar. Look at Example 3 in 12.5. I'll take 20 as an example.

20. Ans: $P(1,0,-1), Q(0,3,0)$. The line can be determined by P and $v = \overrightarrow{PQ} =< -1,3,1 >$. The line is $x = 1-t, y = 3t, z = -1+t$. You can find P and Q. For this problem, we can do directly. From the picture, you can see clearly that the segment is the set of all the terminal points of vectors whose initial points are all P and length is less than or equal to PQ. The segment is thus $P + tv$, where $0 \le t \le 1$. Hence it is $x = 1-t, y = 3t, z = -1+t$, $0 \le t \le 1$. The direction is obviously from P to Q.

27. Ans: The point, if it exists, must satisfy both equations. Hence we have $2t+1 = s+2, 3t+2 = 2s+4$, which means $t = 0, s = -1$. We must check whether the z's are equal. $4t+3 = 4*0+3 = 3$ and $-4s-1 = 4-1 = 3$. They are equal. The point hence is $(1,2,3)$. Two vectors are $v_1 =< 2,3,4 >, v_2 =< 1,2,-4 >$. One normal vector is $v_1 \times v_2 =< -20,12,1 >$. The plane is $-20x+12y+z = -20*1+12*2+1*3 = 7$

28 is similar to 27. Omitted.

39 and 42 are similar, and I'll take 42 as an example.

42. Ans: The distance is $d = PS * \cos\theta = |\overrightarrow{PS} \cdot n|/|n|$.
One normal vector is $n =< 2,1,2 >$. We can find any point on the plane. For example $(1,0,1)$ or $S(2,0,0)$. I'll use $S(2,0,0)$. Then the distance is $| < 0,-2,-3 > \cdot < 2,1,2 > |/| < 2,1,2 > | = 8/3$

45. Ans: We can see that their normal vectors are parallel to each other, and hence they are parallel to each other. The distance between them is equivalent to the distance between one point on one plane to the other plane. A point on the first plane is $P(1,0,0)$. We should find the distance between P and the second plane. Find a point on the second plane $S(10,0,0)$. The normal vector is $n =< 1,2,6 >$. The distance is $d = | < 9,0,0 > \cdot < 1,2,6 > |/| < 1,2,6 > | = 9/\sqrt{41}$

46. Ans: You can check that the direction vector of the line dot the normal vector of the plane is zero, so they are parallel. Then the distance is the distance between a point on the line to the plane. A point on the line is $P(2,1,-1/2)$. A point on the plane is $S(10,0,0)$. The normal vector is $n =< 1,2,6 >$. The distance is the same as the answer in 45 that is $9/\sqrt{41}$. Actually, the line is exactly lies in the first plane in 45.

61 and 62 are similar. I'd like to take 62 as an example.

62. Ans:The first step is to find vectors parallel to them.
$v_1 =< 2,-1,3 >, v_2 =< -1,3,1 >, v_3 =< 2,-1,3 >$. You can see that $v_1 = v_3$ and v_1 is

not parallel to v_2. Then you can check that $L1$ and $L3$ are not the same line, because $(1,-1,0)$ is on the first line, but it's not on the latter. We conclude that $L1$ and $L3$ are parallel to each other. Look at $L1$ and $L2$. We should check whether they have a common point. Letting $1+2t=2-s, -1-t=3s$ $s=-3/5, t=4/5$. Then $3t \neq 1+s$. We conclude that $L1$ and $L2$ are skew. When $s=1, r=-2$, the points on $L2$ and $L3$ are equal. We conclude that they intersect.

70. We must let the normal vector of the second plane be perpendicular to the first plane. The normal vector of the first plane is $n_1 =< 2,3,1 >$. We can let $n_2 =< 3,-2,0 >$. Actually, we have infinitely many normal vectors. The plane is through $(0,0,0)$. The equation is $3x-2y=0$. You can use $n_2 =< s,t,-2s-3t >$ for any s,t.

Section 13.1 1,4,9,10,15,21,24,27,28,33,34,37,40,41,45

1. Ans:$x=t+1, y=t^2-1$. $t=x-1$, and the equation is $y=(x-1)^2-1=x^2-2x$. The velocity is the derivative of the position vector, which is $v=i+2tj$. $v(1)=i+2j$. The acceleration is the derivative of the velocity, which is $a=2j$. $a(1)=2j$.

4 can be solved with similar method. The answer is $x^2+(y/3)^2=1$. $v(0)=6j$. $a(0)=-4i$

9 and 10 are similar. I'd like to take 10 as an example.

10. Ans:The velocity is $v(t)=i+\sqrt{2}tj+t^2k$ and the acceleration is $a=\sqrt{2}j+2tk$. Then $v(1)=i+\sqrt{2}j+k =< 1,\sqrt{2},1 >$. The speed is 2 and the direction is $< 1/2, \sqrt{2}/2, 1/2 >$

15. Ans:$v=3i+\sqrt{3}j+2tk$ and $a=2k$. When $t=0$, $v =< 3,\sqrt{3},0 >, a =< 0,0,2 >$. The angle is $\cos^{-1}(0)=\pi/2$

21. Ans:$i/4+7j+3k/2$

24. Ans:$i+\ln 2j+3k/4$

27 and 28 are similar.

28. Ans: $r(t)=90t^2i+(90t^2-16t^3/3)j+\overrightarrow{C}$. We can determine that $\overrightarrow{C}=100j$

33 and 34 are similar. I'd like to take 34 as an example.

34. Ans: The velocity is $v =< 2\cos t, -2\sin t, 5 >$. When $t_0=4\pi$, $v =< 2,0,5 >$. The point is $P(0,2,20\pi)$. The line is thus $x=2t, y=2, z=20\pi+5t$

37 is easy but needs a lot of typing. I'd like to omit it here.

40. Ans: The acceleration is the second derivative of the position vector. Hence $r''(t)=2i+j+k$. The initial point is $r(0)=i-j+2k$. We need to find the velocity vector at $t=0$. Since it moves along the vector $(3,0,3)-(1,-1,2) =< 2,1,1 >$. The direction is $< 2,1,1 > /\sqrt{6}$. The velocity is $v=2*(< 2,1,1 > /\sqrt{6}) =< 4,2,2 > /\sqrt{6}=r'(0)$

Solve this differential equation, you'll have

$r(t)=(t^2+4t/\sqrt{6}+1)i+(t^2/2+2t/\sqrt{6}-1)j+(t^2/2+2t/\sqrt{6}+2)k$

41. Ans:First of all, we need to find the direction at $(2,2)$. We need to find a vector which is parallel to the tangent line. One vector is $< 1, f'(x) >$, which is $< 1, (\sqrt{2x})' > =< 1, 1/\sqrt{2x} >$. It is $< 1, 1/2 >$. The velocity is thus $5* < 1,1/2 > /| < 1,1/2 > | =< 2\sqrt{5}, \sqrt{5} >$

You can also use other parametrization, for example $r=2s^2i+2sj$, and then then tangent line is parallel to $< 4s, 2 >$. When $r=2i+2j$, $s=1$, and then the vector is $< 4,2 >$, which is also parallel to $< 1,1/2 >$

45. Ans: This is easy $d|v|^2/dt = d(v\cdot v)/dt = 2v\cdot(dv/dt)=0$, and the conclusion follows.