Math 222 Keys and Hints for HW13

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I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 12.5

2,3,6,7,8,9,10,11,21,22,23,24,25,26,31,33,35,36,47,48,57,58,59,65,67,72

2. Ans: $v = \overrightarrow{PQ} =< -2, -2, 2 >$. The line is $x = 1 - 2t, y = 2 - 2t, z = -1 + 2t$

3 is similar to 2 and is omitted.

6. We can use the vector of the second line, which is $v =< 2, -1, 3 >$. The line is $x = 3 + 2t, y = -2 - t, z = 1 + 3t$

7. Ans:$x = 1, y = 1, z = 1 + t$

8. Ans: The vector that is perpendicular to the plane is $v =< 3, 7, -5 >$. The line is $x = 2 + 3t, y = 4 + 7t, z = 5 - 5t$

9 is similar to 8, so I omit it here.

10. Ans: $u \times v = -2i + 4j - 2k$. The line is $x = 2 - 2t, y = 3 + 4t, z = -2t$

11. Ans: $x = t, y = 0, z = 0$ 12. Ans: $x = 0, y = 0, z = t$

21. Ans: $3 * x + (-2) * (y - 2) + (-1) * (z + 1) = 3x - 2y + 4 - z - 1 = 0$, namely $3x - 2y - z = -3$

22. Ans: Choose vector $n =< 3, 1, 1 >$. The plane is $3x + y + z = 3 * 1 + 1 * (-1) + 1 * 3 = 5$

23. Two methods. One is plug the three points in to the equation $Ax + By + Cz = D$ and then let one of A, B, C equal to 1. You can get the answer.
The other one is calculate the vector. $< 1, -1, 3 >, < -1, -3, 2 >$ and the cross product of them is $< 7, -5, -4 >$. The plane is $7x - 5y - 4z = 7 * 1 - 5 * 1 - 4 * (-1) = 6$

24 is similar to 23, omitted.

25. Ans: $n =< 1, 3, 4 >$. The plane is $x + 3y + 4z = 1 * 2 + 3 * 4 + 4 * 5 = 34$

26. Ans: $n =< 1, -2, 1 >$. $x - 2y + z = 1 * 1 + (-2) * (-2) + 1 * 1 = 6$

31. Ans: One normal vector is $n =< 2, 1, -1 > \times < 1, 2, 1 >=< 3, -3, 3 >$. The plane is $3x - 3y + 3z = 3 * 2 - 3 * 1 + 3 * (-1) = 0$, which is equivalent to $x - y + z = 0$

33. Ans: $v =< 4, -2, 2 >$. One point on the line is $(0, 0, 0)$. The distance is $| < 0, 0, -12 > \times < 4, -2, 2 > /| < 4, -2, 2 > || = | < -24, -48 > /\sqrt{24}| = 2\sqrt{30}$

35 and 36 are similar to 33. Omitted here.

47 and 48 are similar. I'll take only one as an example.

47. Ans: Two vectors are $< 1, 1, 0 >$ and $< 2, 1, -2 >$. The angle between the two planes is the acute or right angle. The value of the cosine is the absolute value of the cosine of the angle between these two vectors. $\cos\theta = \frac{2+1-0}{\sqrt{2}*\sqrt{9}} = 1/\sqrt{2}$. The angle is $\pi/4$.

57,58 and 59 are similar. I'll take 57 as an example.

57. Ans: The two normal vectors are $< 1, 1, 1 >$ and $< 1, 1, 0 >$. The vector which is parallel to the line is $< 1, 1, 1 > \times < 1, 1, 0 >=< -1, 1, 0 >$. One point on the line is

$(0, 2, -1)$. (I just assume $x = 0$.). The line is $x = -t, y = 2 + t, z = -1$. Can you understand why my answer looks a little different from the answer in the book?
65. For xy-plane, $z = 0$. Then $t = 0$, and $x = 1, y = -1$. The line meets the xy-plane at $(1, -1, 0)$. Similarly it meets the yz-plane at $(0, -1/2, -3/2)$ and meets xz-plane at $(-1, 0, -3)$.
67. One vector which is parallel to the line is $< -2, 5, -3 >$ and one vector which is perpendicular to the plane is $< 2, 1, -1 >$. The dot product between them is $-4 + 5 + 3 \neq 0$. Hence the line is not parallel to the plane.
72. Yes. It's possible. We can find the direction vectors for these two lines. Since they are not parallel. The cross product is none zero and thus perpendicular to both vectors and hence perpendicular to both lines. Further more, we can even find a line, which intersects with both lines and perpendicular to both lines. Can you think of a way to find it? How about the two lines are parallel to each other?