Math 222 Keys and Hints for HW12

By Lei   November 30, 2010

I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 12.3

1–5,16,18,19,22,30,32

1–5 are similar. I'll take only one as an example. Remember $\cos\theta = \frac{u\cdot v}{|u||v|}$. The scalar component of u in the direction of v is $|u|\cos\theta = u\cdot v/|v|$ and $proj_v u = \frac{u\cdot v}{|v|^2}v$

1. Ans: $v\cdot u = 2*(-2)+(-4)*4+(\sqrt{5})*-\sqrt{5} = -4-16-5 = -25$. $|v| = |u| = 5$. $\cos\theta = -1$. Scalar comp -5 and vector is $u = -2i+4j-\sqrt{5}k$

16. Ans:I can't draw the picture here, so I just describe it. I'd like to take the x-axis in the south, y-axis to the east and z-axis up. Since we only need to determine the angle between two directions, we only need to find the unit vector in those two directions. I'd like to take one vector to the south-down and one vector east-up. 20% grade means if I go to the north for 100 miles, I'll rise for 20 miles This means the absolute value of the slope is 0.2. We can write the first unit vector out as $u_1 =< \frac{5}{\sqrt{26}}, 0, -\frac{1}{\sqrt{26}} >$. Similarly, the second unit vector is $u_2 =< 0, \frac{10}{\sqrt{101}}, \frac{1}{\sqrt{101}} >$. The angle is $\theta = \cos^{-1}(-\frac{1}{\sqrt{101*26}}) = \pi - \cos^{-1}(\frac{1}{\sqrt{101*26}})$

18 and 19 are similar. I'll only take 18 as an example.

18. Ans:We know the projection of u onto v is parallel to v and the difference of u and $proj_v u$ is orthogonal to v. $proj_v u = \frac{u\cdot v}{|v|^2}v = \frac{1}{2}(i+j)$. We have the answer as $u = \frac{1}{2}(i+j) + (\frac{1}{2}(j-i)+k)$

22. Ans:We have two methods to do this.

First: $\overrightarrow{CA} = -u-v$ and $\overrightarrow{CB} = u-v$. We have $\overrightarrow{CA}\cdot\overrightarrow{CB} = -u\cdot u + u\cdot v - v\cdot u + v\cdot v = 0$. The conclusion holds.

Second: Take x-axis as the line AB and y-axis as the line throgh O and up. Then $A(-r,0), B(r,0)$ and $C(r\cos\alpha, r\sin\alpha)$. Write out the two vectors and you can check the dot product is zero.

30. Ans:Find all the vectors such that the angle between them is equal to or bigger than $pi/2$. Assume the line through the origin and perpendicular to v is L. The region is that one on the left of L or on the L.

32. Ans:No, we can't get the answer. For example $< 1,0 > \cdot < 1,0 > = < 1,0 > \cdot < 1,1 >$, however $< 1,0 > \neq < 1,1 >$. Can you give a geometric explanation of why this happens? Extra question:if this equation holds for all u, what's the answer?

Section 12.4

1–7,15,16,21,22,23,27,28,29,31,37,41,42

1–7 are similar. I'd like to take only one as an example.

1. Ans: $u =< 2, -2, -1 >, v =< 1, 0, -1 >$.

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -2 & -1 \\ 1 & 0 & -1 \end{vmatrix} = i \begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} = 2i + j + 2k \quad (1)$$

The length is $\sqrt{4 + 1 + 4} = 3$ and the direction is $< 2/3, 1/3, 2/3 >$.
As for $v \times u$, it's just $-u \times v$ and you can get the answer easily from $u \times v$.
15 and 16 are similar. I'd like to take 16 as an example.
16. Ans:$P(1, 1, 1), Q(2, 1, 3), R(3, -1, 1)$, so $u = \overrightarrow{PQ} =< 1, 0, 2 >$ and $v = \overrightarrow{PR} =< 2, -2, 0 >$.
We have:

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = 4i + 4j - 2k \quad (2)$$

The area of the triangle is half of the length of this vector that is $\frac{1}{2}\sqrt{16 + 16 + 4} = 3$. The unit vector is the normal vector, that is $< 4/3, 4/3, -2/3 >$
21 and 22 are similar, and I'd like to take 21 as an example.
21. Ans: I'll only calculate $(u \times v) \cdot w$. $u =< 2, 1, 0 >, v =< 2, -1, 1 >, w =< 1, 0, 2 >$.

$$(u \times v) \cdot w = \begin{vmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = 2 * (-2) - 1 * (4 - 1) + 0 = -7 \quad (3)$$

Hence, the volume is 7.
23. Ans: $u =< 5, -1, 1 >, v =< 0, 1, -5 >, w =< -15, 3, -3 >$. Since $u = -3w$, u and w are parallel to each other. Since $u \cdot v = 0$ and u are parallel to w, we conclude at last that u and w are parallel to each other and they both are perpendicular to v.
27. Ans: $|u| = \sqrt{u \cdot u}$ is always true, because $u \cdot u = |u||u| \cos 0 = |u|^2$. Since we have this, b can't be always true.
c and d are obviously true. e is not always correct. f,g and h are always true. Reasons are omitted here.
28. a, b, c, d, e, f, g and h are all true.
29. a. $\frac{u \cdot v}{|v|^2} v$
b. $u \times v$
c. $(u \times v) \times w$
d. $|(u \times v) \cdot w|$
31. Ans: a and c make sense while b and d don't make sense. This is easy but very important.Think of the reasons yourself!
37. Ans: After drawing the picture, we can see that two edges are AB and AD.
$u = \overrightarrow{AB} =< 3, -2 >$ and $v = \overrightarrow{AD} =< 5, 1 >$. The area of the parallelogram is:

$$\left| \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} \right| = 13 \quad (4)$$

41 and 42 are similar. I'll take 42 as an example.
42. Ans: $u = \overrightarrow{AB} =< 16, -5 >, v = \overrightarrow{AC} =< 4, 4 >$. The area of the triangle is:

$$A = \frac{1}{2} * | \begin{vmatrix} 16 & -5 \\ 4 & 4 \end{vmatrix} | = \frac{1}{2} * (64 + 20) = 42 \tag{5}$$