Math 222 Keys and Hints for HW10

By Lei November 11, 2010

I HAVE NO ANSWERS AND THE FOLLOWINGS ARE WHAT I GOT. I FOUND THE CALCULATION WAS NOT FUN AND I MIGHT MAKE MISTAKES. BE CAREFUL WHEN YOU READ WHAT I WROTE.

Section 17.2

3,4,8,9,12,29,30,31,52,58

For an inhomogeneous second order, linear differential equation with constant coefficients, the general solution can always be written as any particular solution y_p plus the general solution to the complementary equation y_c that is the corresponding homogeneous equation.

3. Ans:$y'' - y' = \sin x$. The complementary equation is $y'' - y' = 0$ and the auxiliary equation is $r^2 - r = 0$. $y_c = C_1 e^x + C_2$. $\pm i$ is not the root, so we can try $y_p = A\sin x + B\cos x$. $y_p = \frac{1}{2}(\cos x - \sin x)$. The general solution is $y(x) = C_1 e^x + C_2 + \frac{1}{2}(\cos x - \sin x)$

4. Ans:Complementary equation $y'' + 2y' + y = 0$ and $y_c = C_1 e^{-x} + C_2 x e^{-x}$. Since the coefficient of y is not zero, we can try $y_p = Ax^2 + Bx + C$, and then we can decide $y_p = x^2 - 4x + 6$. The general solution $y(x) = C_1 e^{-x} + C_2 x e^{-x} + x^2 - 4x + 6$

8. Ans:The corresponding complementary equation $y'' + y = 0$. $y_c = C_1 \cos x + C_2 \sin x$. We have two force terms on the right hand side. We can find y_p one by one (Why? Justify yourself! Maybe I'll give this as a bonus problem in quiz). For $2x$ we only need $2x$. For $3e^x$, since 1 isn't a root of the auxiliary equation, we can use Ae^x to try and get $3e^x/2$. We finally get $y(x) = C_1 \cos x + C_2 \sin x + 2x + 3e^x/2$.

9. Ans:The complementary equation $y'' - y = 0$. $y_c = C_1 e^x + C_2 e^{-x}$. For x^2, we have $-x^2 - 2$. Since 1 is a single root of the auxiliary equation, we can try Axe^x and we get $xe^x/2$. Finally, we have $y(x) = C_1 e^x + C_2 e^{-x} - x^2 - 2 + xe^x/2$

12. Ans:Complementary: $y'' + 3y' + 2y = 0$ $y_c = C_1 e^{-x} + C_2 e^{-2x}$. For $-x$, we can try $Ax + B$ and we get $-\frac{1}{2}x + \frac{3}{4}$. Since both -2 and -1 are single roots, we can try $A_1 x e^{-x}$ and $A_2 x e^{-2x}$ respectively. We can have xe^{-x} and $-xe^{-2x}$. $y(x) = C_1 e^{-x} + C_2 e^{-2x} + xe^{-x} - xe^{-2x}$

29. Ans:Complementary $y'' - 5y' = 0$. $y_c = C_1 e^{5x} + C_2$. Since 5 is a single root and the force term is xe^{5x}, we can try $Ax^2 e^{5x} + Bxe^{5x}$.
$y_p'' = 2Ae^{5x} + 20Axe^{5x} + 25Ax^2 e^{5x} + 10Be^{5x} + 25Bxe^{5x}$.
$y_p' = 2Axe^{5x} + 5Ax^2 e^{5x} + Be^{5x} + 5Bxe^{5x}$. $y_p = \frac{1}{10}x^2 e^{5x} - \frac{1}{25}xe^{5x}$
$y = C_1 e^{5x} + C_2 + \frac{1}{10}x^2 e^{5x} - \frac{1}{25}xe^{5x}$

30. Ans: C:$y'' - y' = 0$ $y_c = C_1 e^x + C_2$. Since $\pm i$ is not the root and the right hand side is the sin, cos with frequency 1, we can try $A\sin x + B\cos x$ directly.We have $-\sin x$.
$y = C_1 e^x + C_2 - \sin x$

31. Ans:C: $y'' + y = 0$ $y_c = C_1 \cos x + C_2 \sin x$. Now, since $\pm i$ are the roots. For sin and cos on the right, we must use x multiplying the corresponding term. Namely, we should try

$y_p = Ax\cos x + Bx\sin x$. $y_p = -\frac{1}{2}x\cos x + x\sin x$. The general solution is
$y = C_1\cos x + C_2\sin x - \frac{1}{2}x\cos x + x\sin x$

52. Ans:$y_c = C_1\cos x + C_2\sin x$. 2 is not the root and we have $y_p = e^{2x}/5$. Hence, the general solution $y = C_1\cos x + C_2\sin x + e^{2x}/5$. We then use the initial values to determine the coefficients. $y(0) = 0$ implies $C_1 = -1/5$. $y'(x) = -C_1\sin x + C_2\cos x + 2e^{2x}/5$. We have $C_2 + 2/5 = 2/5$, so $C_2 = 0$. Finally, we have $y(x) = -1/5\cos x + e^{2x}/5$

58. Ans:You **must** verify yourself that the given expression is the particular solution. I'll omit it here. $y_c = C_1e^x + C_2xe^x$ and hence the general solution is
$y(x) = C_1e^x + C_2xe^x + xe^x\ln x$. We then use the two values to determine the coefficients.
$y(1) = C_1e + C_2e + 0 = e$, which means $C_1 + C_2 = 1$.
$y'(x) = (C_1 + C_2)e^x + C_2xe^x + e^x\ln x + xe^x\ln x + e^x$. $y'(1) = (C_1 + 2C_2)e + e = 0$, which means $C_1 + 2C_2 = -1$. We have $y(x) = 3e^x - 2xe^x + xe^x\ln x$

Section 17.3

1,7,8,22

1. Ans: Set positive direction to be downward. Let y be the displacement away from the equilibrium. We can ignore the gravity if we consider the displacement from the equilibrium, since $mg = ks$ as in Sec 17.3. The instantaneous velocity is dy/dt, and the resistance, or the friction is $-dy/dt$ as the problem says. The resultant force is $-y - dy/dt$.
$mg = G$ implies $m = G/g = 16/32 = 1/2\ lb * sec^2/ft = 1/2slug$. Newton's law says:
$\frac{1}{2}\frac{d^2y}{dt^2} = -y - \frac{dy}{dt}$
$y(0) = 2$ and $y'(0) = 2$

7. Ans:The problem is the same as 1. Here,we must solve this equation:
This is a homogeneous second order differential equation with constant coefficients.
$\frac{1}{2}r^2 + r + 1 = 0$. Hence $y(t) = e^{-t}(C_1\cos t + C_2\sin t)\ ft$.
$C_1 = 2$ and $C_2 = 4$. $y(\pi) = e^{-\pi}(-2) < 0$, so the mass is above the equilibrium and the distance is $2e^{-\pi}\ ft$

8. Ans:Since $mg = ks$, we know the spring constant is $k = 8lb/4ft = 2\ lb/ft$.
$m = mg/g = 8/32 = 1/4\ slug$. Also let the positive direction be downward and y be the displacement from equilibrium. We can have the equation:
$\frac{1}{4}y'' = -2y - 1.5y'$, namely $y'' + 6y' + 8y = 0$.
$y(0) = -2$ (above) and $y'(0) = 3$.
$y(t) = C_1e^{-2t} + C_2e^{-4t}$.
$y(t) = -\frac{5}{2}e^{-2t} + \frac{1}{2}e^{-4t}$ and $y(2) = \ldots$

22. Ans:The equation can be written as $10y'' = -140y - 90y' + 5\sin t$. $y(0) = 0$ and $y'(0) = -1$
$y = C_1e^{-2t} + C_2e^{-7t} + \frac{13}{500}\sin t - \frac{9}{500}\cos t$. Then you calculate the following, and I'd like to stop here.