Math 221 Keys and Hints for HW1
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17. \[ \int_0^{\ln^2} 2xe^x \, dx \]
   Ans: 1.
   Hint: Noticing that \( 2x \) is the derivative of \( x^2 \) and the remaining part is a function of \( x^2 \), we can use the substitution \( u = x^2 \) to get the antiderivative.

22. \[ \int \frac{\ln x}{x^2} \, dx \]
   Ans: \( \frac{\ln x}{\ln 2} + C \)
   Hint: Noticing that \( 1/x \) is the derivative of \( \ln x \), we can let \( u = \ln x \)

29. \[ \int \frac{2s}{\sqrt{1-s^2}} \, ds \]
   Ans: \( \arcsin(s^2) + C \)
   Hint: \( u = s^2 \)

37. \[ \int_1^2 \frac{8}{x^2+x^2} \, dx \]
   Ans: \( 2\pi \)
   Hint: Completing the square, we have \( (x - 1)^2 + 1 \) in the denominator, so the antiderivative is \( 8 \arctan(x - 1) + C \).

48. \[ \int \frac{x^2}{x^4+1} \, dx \]
   Ans: \( x - \arctan x + C \)
   Hint: Reducing the improper fraction, we have \( 1 - \frac{1}{1+x^2} \).

58. \[ \int \frac{1}{\cos x} \, dx \]
   Ans: \( -\cot x + \frac{1}{\sin x} + C \) Or equivalently, \( \tan(x/2) + C \)
   Hint: For the first, multiply \( 1 - \cos x \) on the top and the bottom. You can also write \( \frac{1}{\sin x} \) as \( \csc x \), but I don’t like csc. The second answer comes from that \( 1 + \cos x = 2\cos^2(x/2) \) and \( u = x/2 \).

77. \[ \int \frac{6}{\sqrt{y(1+y)}} \, dx \]
   Ans: \( 12 \arctan(\sqrt{y}) + C \)
   Hint: Notice that \( \frac{1}{\sqrt{y}} \) is the derivative of \( 2\sqrt{y} \), so \( u = \sqrt{y} \).