September 2009 Math 222 I. M. Isaacs

Sample 6th Week Exam Problems

All of these problems have appeared on Isaacs' exams.

1. Integrate. (a) $\displaystyle\int_0^{1/2} \frac{x^2\,dx}{\sqrt{1-x^2}}$ (b) $\displaystyle\int \frac{x\,dx}{x^2+2x+2}$ (c) $\displaystyle\int \frac{dx}{(x-1)(x+1)(x+2)}$
d) $\displaystyle\int \frac{dx}{\cos^4(x)}$ (e) $\displaystyle\int \frac{(3x+1)\,dx}{\sqrt{3x^2+2x+1}}$

2. For each of the following, either compute the value or write DIVERGES.
(a) $\displaystyle\int_1^\infty \frac{dx}{x^2}$ (b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$ (c) $\displaystyle\int_0^\infty \sin(x)\,dx$ (d) $\displaystyle\int_{-1}^1 \frac{dx}{x^3}$

3. Integrate. (a) $\displaystyle\int x^2 e^{x^3}\,dx$ (b) $\displaystyle\int \sin^2(2x)\,dx$ (c) $\displaystyle\int x^4 \ln(x)\,dx$ (d) $\displaystyle\int \sin^3(x)\,dx$

4. Solve these initial value problems.
(a) $\dfrac{dy}{dx} - \dfrac{y}{2x} = x \quad y(3) = 9$ (b) $\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx} + e^x \quad y(0) = 1 = y'(0).$

5. Find the general solution for $y'' - 4y' + 13y = 0$.

6. Write the general solution for each of these.
(a) $y'' + 6y' + 9y = e^x$ (b) $y'' + 4y = x^2$.

7. Use Simpson's rule with four intervals to find an approximation for $\ln(3)$. HINT: Start by writing a simple integral whose value is $\ln(3)$. Work with fractions, not decimals.
NOTE: $\ln(3) \approx 1.098612289$. You can use this to check that your answer is reasonable.

8. I am trying to evaluate $I = \displaystyle\int_{-\sqrt{2}}^{-1} \frac{x^2\,dx}{\sqrt{2x^2-1}}$. I decide to try a trig substitution, and I obtain a definite integral involving trig functions. What integral do I get? DO NOT EVALUATE.

9. Integrate.
(a) $\displaystyle\int \frac{\sin(x)\,dx}{\sqrt{1+\cos(x)}}$ (b) $\displaystyle\int \left(\frac{1+x}{x}\right)^2 dx$ (c) $\displaystyle\int \tan^3(x)\sec(x)\,dx$ (d) $\displaystyle\int_1^e \frac{dx}{x + x\ln(x)}$

10. Integrate. (a) $\displaystyle\int_0^{\pi/9} \tan(3x)\,dx$ (b) $\displaystyle\int \frac{dt}{4t^3 - t}$ (c) $\displaystyle\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x}\,dx$ (d) $\displaystyle\int_1^4 e^{\sqrt{x}} dx$

11. For each of the following integrals, either compute the value, or write DIVERGENT.
(a) $\displaystyle\int_{-1}^2 \frac{dx}{x^3}$ (b) $\displaystyle\int_0^\infty xe^{-x}\,dx$ (c) $\displaystyle\int_{-1}^2 \frac{dx}{e^x}$ (d) $\displaystyle\int_0^\infty \frac{x\,dx}{x+1}$

12. Integrate. (a) $\displaystyle\int_0^\pi \sin^2(x/3)\,dx$ (b) $\displaystyle\int \frac{x^3\,dx}{x^2+5}$ (c) $\displaystyle\int \frac{x\,dx}{x^2+5x+6}$ (d) $\displaystyle\int_1^{e^2} \ln(\sqrt{x})\,dx$
(e) $\displaystyle\int_0^{\pi/2} \frac{\sin(x)\,dx}{(3-2\cos(x))^2}$

13. Integrate. (a) $\displaystyle\int \frac{x^5\,dx}{x^3+1}$ (b) $\displaystyle\int x^2 e^x\,dx$ (c) $\displaystyle\int \frac{dx}{(4+5x^2)^{3/2}}$ (d) $\displaystyle\int \sec^6(x)\,dx$

14. Decide whether or not this integral has a meaningful value. If it does, compute the value and if not, explain why not: $\int_{-1}^{2} \frac{dx}{\sqrt{|x|}}$.

15. Integrate (a) $\int \sec^4(x)\,dx$ (b) $\int \frac{x^7+x^3}{x^4-1}\,dx$ (c) $\int \frac{(2x+3)\,dx}{4x^2+4x+5}$

(d) $\int_0^2 (4-x^2)^{3/2}\,dx$ (e) $\int \frac{\cos(x)\,dx}{\sin^2(x)-3\sin(x)+2}$ (f) $\int x\sin(\ln(x))\,dx$

16. State which of the following are DIVERGENT improper integrals.

(a) $\int_0^\infty \cos(x)\,dx$ (b) $\int_0^\infty \frac{dx}{2+x^2}$ (c) $\int_{-1}^1 \frac{dx}{\sqrt[3]{x}}$ (d) $\int_0^\pi \tan(x)\,dx$ (e) $\int_0^1 \frac{x\,dx}{x^2-1}$

17. Integrate. (a) $\int_0^1 \frac{x+1}{x^2+2x-4}dx$. (b) $\int x\sec^2(x)\,dx$. (c) $\int \cos^4(x/4)\,dx$.

18. Integrate. (a) $\int_0^\pi \sin^2(x/3)\,dx$. (b) $\int \frac{x^3\,dx}{x^2+5}$. (c) $\int \frac{x\,dx}{x^2+5x+6}$.

19. Find general solutions. the next higher power of x as a factor.

P 17-12. (a) $y''+y'-2y=4x$ (b) $y'''-2y''+y'=1$ (c) $y''-2y'+5y=0$.

20. For each of these initial value problems, find the indicated value of y.

(a) $y'=(x^2+y)/x$ $y=2$ when $x=1$. Find y when $x=2$.

(b) $y''=(yy')^3$ $y=1$ and $y'=-4$ when $x=0$. Find y when $x=1$.

21. The graph of the function $y=f(x)$ for $x\geq 0$ has the property that the normal line at each point (x,y) on the curve crosses the y-axis at $(0,y/2)$. The curve crosses the y-axis at $(0,4)$. Where does the curve cross the x-axis?

22. Solve these initial value problems.

(a) $xy'-y=x^3$ $y(2)=6$. (b) $y'=(x+y)/x$ $y(1)=1$.

23. Solve this initial value problem. $y''=8(y^3+y)$. $y(1)=1$, $y'(1)=4$.

24. Let L be the linear differential operator given by the formula $L(y)=x^2y''-2xy'+2y$.

(a) Compute $L(x)$, $L(x^2)$ and $L(x^3)$. (b) Find the general solution for $x^2y''-2xy'+2y=4x^3$.

25. Integrate. (a) $\int_0^1 \frac{x+1}{x^2+2x-4}dx$. (b) $\int x\sec^2(x)\,dx$. (c) $\int \cos^4(\frac{x}{4})\,dx$.

26. Integrate. $\int_2^4 \frac{\sqrt{x^2-4}}{x}\,dx$.

27. Use Simpson's rule with $n=4$ to obtain a numerical approximation for $\int_1^3 \frac{dx}{1+x}$.

28. Integrate. (a) $\int_1^{\sqrt{2}} \frac{dx}{\sqrt{2x^2-1}}$. (b) $\int x^2\ln(x)\,dx$.

29. Integrate. (a) $\int \frac{4x^4+1}{4x^3-x}\,dx$. (b) $\int \sin(\ln(x))\,dx$.