Why does an airplane stay up in the air?

(References are to the books by Acheson (A) and Chorin & Marsden (CM). See especially A, pp. 18–23 and pp. 158-59 for a version of this. Warning: There are several qualifications to this story. See the remarks at the end.)

Why does an airplane stay up in the air?
Because there is lift on the wing, an upward force counteracting the downward force of gravity.

Why is there lift on the wing?
Because the air pressure just above the wing is much lower than atmospheric pressure, whereas the pressure just below the wing is a bit greater than atmospheric pressure, giving a net upward force. (See A p. 18.) Bernoulli’s Law tells us that, equivalently, the air speed is greater above the wing and less below (A pp. 9-10, CM pp. 15, 48).

Why is the pressure lower above the wing and greater below?
Because there is (negative) circulation around the wing. That is, the velocity field has negative circulation around a curve in the plane of a cross-section of the wing. This circulation gives the qualitative difference above and below the wing. In fact the Kutta-Joukowski Theorem tells us the lift is proportional to the circulation (A p. 143, CM p. 53).

Why is there circulation around the wing?
This is a subtle point. There could be flow with no circulation, or with an arbitrary value, but in most cases there would be a singularity in the velocity at the trailing (back) edge of the wing. We believe the right amount of circulation is generated so that there is no singularity. This is the Kutta condition. (A p. 20; a simplified model is worked out explicitly in A pp. 137-40.)

How is this circulation generated?
We are thinking of the airplane in steady flight. As it was reaching the steady state, positive circulation was set up behind the plane. Kelvin’s Circulation Theorem (A p. 157, CM p. 21) tells us circulation is conserved, so there is compensating negative circulation around the wing.

Where does the positive circulation behind the plane come from?
It is generated by vorticity (the curl of the velocity) which is shed behind the wing (A p. 1). For the relation between circulation and vorticity, see CM p. 22.

(over)
Why is vorticity shed behind the wing?

Ah, now we are really getting somewhere. Until now we have assumed there is no viscosity. In fact, D’Alembert’s Paradox (A p. 147, CM p. 57) tells us there could not be lift on a 3D object if there were no viscosity. For aircraft flight the Reynolds number is large. (I.e., the nondimensional coefficient of viscosity is small.) For such flow, in most of the region the effect of viscosity is insignificant, but it is important in a thin boundary layer near the object, and in certain regions where this layer separates from the boundary (A Sec. 8.1-8.3, CM Sec. 2.2). The airplane wing has a boundary layer which separates at the back of the wing and leaves a trail of vorticity behind the wing. Mathematical representation of the boundary layer is one of the most important and difficult cases of a singular perturbation.

**Remarks:** Until the end of this story, we were assuming two-dimensional, incompressible flow with no viscosity. 3D effects matter. especially at the tip of the wing. The air flow is approximately incompressible if the Mach number is much smaller than one, i.e., the air speed is much smaller than the speed of sound. That is true of small (private) planes, and earlier planes, but not the ones you get on at RDU. For those we need the equations of compressible flow. We noted the importance of viscosity above. Despite these qualifications, this explanation is well supported by experience, experiment, and more detailed studies.

Prandtl observed and explained the boundary layer and its separation about 1905. Hardly anyone paid attention to him, but hardly anyone realized how the world was about to change.

The flight of birds and insects is more complicated than that of airplanes since airplanes don’t flap their wings. There are many connections between fluid mechanics and biology. There is a great book by Steven Vogel, of Duke’s biology dept., called Life in Moving Fluids.