Quiz 1

Suppose $L_1$ is the line of intersection of planes $x + y + z = 5$ and $3x - y = 4$. $L_2$ is the tangent line of $r(t) = \langle t, t^2, t^3 \rangle$ at $t = 1$ (This means that $L_2$ passes through $r(1)$ and is parallel with the velocity at $t = 1$). Find the equation of the plane that contains $L_2$ and is parallel with $L_1$.

For $L_1$: $x + y + z = 5$ and $3x - y = 4$. Setting $x = 1$, we find $y = 3 - 4 = -1$ and $z = 5 - x - y = 5$. Hence, $(1, -1, 5)$ is on the intersection.

If we set $z = 1$, then $x + y = 4, 3x - y = 4$ and $x = 2, y = 2$. Hence $(2, 2, 1)$ is also on the line. Hence, $v_1 = \langle 2 - 1, 1 - 2, 1 - 5 \rangle = \langle 1, 3, -4 \rangle$ is parallel with $L_1$.

For $L_2$, $r(1) = P(1, 1, 1)$ is on it. $v_2 = r'(1) = \langle 1, 2, 3 \rangle$ is parallel with $L_2$.

Since the plane contains $L_2$, $(1, 1, 1)$ is in the plane. A normal vector is $n = v_1 \times v_2 = \langle 17, -7, -1 \rangle$. The equation is

$$17(x - 1) - 7(y - 1) - (z - 1) = 0.$$ # Remark 1.

For $L_1$, another way for finding a vector parallel with it is to take the cross of the normal vectors of the two planes $\bar{v} = (1, 1, 1) \times \langle 3, -1, 0 \rangle = (1, 3, -4)$