Quiz 1

You score will be at most 10. If you get more than 10 with bonus, I will give you 10 points.

Consider the following two skew lines:

\[ L_1 : x = 1 + t, \quad y = -1 - t, \quad z = 2 - 3t \]
\[ L_2 : \begin{array}{c}
x = \frac{y - 1}{1} = \frac{2 - z}{1}
\end{array} \]

(a). Find two vectors \( \vec{v}_1 \) and \( \vec{v}_2 \) such that \( \vec{v}_1 \) is parallel to \( L_1 \) and \( \vec{v}_2 \) is parallel to \( L_2 \) (4 points).

The second line can be written as \( \frac{x}{1} = \frac{y - 1}{1} = \frac{2 - z}{-3} \). Two possible vectors are:

\( \vec{v}_1 = \langle 1, -1, -3 \rangle, \quad \vec{v}_2 = \langle 1, 1, -1 \rangle \).

(b). Find a plane that contains \( L_1 \) and that is parallel to \( L_2 \) (6 points).

A normal is

\( \vec{n} = \vec{v}_1 \times \vec{v}_2 = \langle 4, -2, 2 \rangle \sim \langle 2, -1, 1 \rangle \).

A point on the plane is \( (1, -1, 2) \) by setting \( t = 0 \) in \( L_1 \). Hence, the equation of the plane is

\[ 2(x - 1) - (y + 1) + 1(z - 2) = 0 \Rightarrow 2x - y + z = 5. \]

(c). Bonus (2 points): find the distance between the two skew lines.

The distance is equal to the distance of a point on \( L_2 \) to the plane we find in Part (b). A point on \( L_2 \) is \( Q(0, 1, 2) \) while a point on the plane in \( P(1, -1, 2) \). The distance is therefore

\[ d = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|} = \frac{|(-1) * 2 + 2 * (-1) + 0 * 1|}{\sqrt{4 + 1 + 1}} = \frac{4}{\sqrt{6}}. \]