Math 212-Midterm 1 (Part 1)
Fall 2017, Lei Li

Name: ________________________________

Instructions: There are 3 problems. This part accounts for 60% of Midterm 1. You have 40 minutes. No calculators are allowed. The bonus will accumulate towards the total score of Midterm 1, but your total score of Midterm 1 will not exceed 100.

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1. (7+7+6)

Suppose \( \vec{a} = 2\vec{i} + \vec{j} + 2\vec{k} \). Vector \( \vec{b} \) is the velocity of \( \vec{r}(t) = (e^t, 1 - t, \cos(t)) \) at \( t = 0 \).

(a). Find two unit vectors that are perpendicular to both \( \vec{a} \) and \( \vec{b} \).

(b). Find a vector \( \vec{a}_\perp \) that is perpendicular to \( \vec{b} \) such that \( \vec{a} - \vec{a}_\perp \) is parallel with \( \vec{b} \).

(c). If the volume of the parallelepiped determined by \( (\vec{a}, \vec{b}, \vec{c}) \) is 1, what is the volume of the parallelepiped determined by \( (2\vec{a} - \vec{b}, \vec{a} - 3\vec{c}, -2\vec{c}) \)? If \( (\vec{a}, \vec{b}, \vec{c}) \) is right handed, how about the latter?
2. \((12+8+5)\)

Assume that the function

\[
z = f(x, y) = \frac{1}{1 + \ln(2x^2 + e^y)} + \arctan(1 + x) + 1
\]

describes the landform of Duke campus where \(z\) represents the height while \((x, y)\) represents the location on map.

(a). Compute \(\nabla f(0, 0)\) and use this to compute the height at location \((x, y) = (0.1, 0.1)\) approximately (use \(\pi = 3.14\)).

(b). Suppose that Jack is running on a trail and the trajectory of Jack on the map (i.e. we view him in the sky so that Jack is moving on a plane) is given by

\[
\vec{r}(t) = (t^2 - 1, t^3 - 1),
\]

where \(t\) is the time. What is the rate of climb (rise over run) of Jack at \(t = 1\)? What is the rising rate of Jack with respect to time at \(t = 1\)? If Jack wants to rise the fastest at \(t = 1\) with the same horizontal speed, in which direction should Jack run?

(c). The function \(z = f(x, y)\) can be regarded as a level set of a function: \(F(x, y, z) = 0\). Find such a function \(F\). Use this fact to find the tangent plane of the function graph at \((0, 0, 2 + \frac{\pi}{4})\).
3. (10+5+bonus)

- Use chain rule to compute \( \frac{\partial z}{\partial x} \) at (0, 0) where \( z = f(\sin(x + 2y)) + xu \). \( f \) is a one-variable function satisfying \( f'(0) = 1 \) and \( u = \sqrt{1 + x + y} \) (for this question, do not plug \( u \) into the formula of \( z \) to compute).

- \( z = f(x, y) \) is determined by \( F(\sin(x + z) + 2z) + e^y + y = 0 \) where \( F \) is some given one-variable function with \( F'(u) > 0 \) for any \( u \). Write out \( f_y \) in terms of \( x, y, z \) and \( F \).

- (Bonus: 4 points) Suppose \( f(x, y) \) satisfies that \( f_{xx}(a, b) - f_{yy}(a, b) = 0 \) for any point \( (a, b) \). Verify that

\[
z = g(u, v) := f(u + v, u - v)
\]

solves the equation

\[
\frac{\partial^2 g(u, v)}{\partial u \partial v} = 0.
\]

where := means the left hand side is defined by the right hand side.