Quiz 6

1. Change the following surface integrals into suitable double integrals through suitable parametrizations:

   - Let $S$ be the portion of $y = \sin(x + z)$ inside $z = x^2 - 2x$ and $z = x$, with the normal pointing along positive $y$ direction.
     \[\int\int_S (y^2 + x^2)\,dS, \quad \int\int_S yz\,dy\,dz.\]
   - \[\int\int_S \vec{F} \cdot \vec{n}\,dS\]
     where $\vec{F} = \langle y, y, x \rangle$ and $S$ is the part of $z = 6 - x^2 - y^2$ above the cone $z = r$ with upward normal.

2. Consider the part of the cone $z = r$ between the two paraboloids $z = 20 - r^2$ and $z = 2 - r^2$, with the normal pointing away from $z$-axis. Let $\vec{F} = \langle x \ln(x^2 + 1) + yz, e^{y^2} - xz, z^{93} \rangle$. Compute the flux
     \[\int\int_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS.\]

Here, I provide three ideas. Choose two ideas to compute and then verify you get the same answer.

   - Use Stokes theorem and then evaluate some line integrals along the two boundary curves. To evaluate the line integrals, you can throw away a certain conservative field (why?)
   - Evaluate the surface integral directly by parametrizing.
   - Use surface independence to change the integral to integrals over two disks. (Why can you apply surface independence? Which two disks?)