Math 212-Lecture 1

Information

• This course contains a lot and you may find the pace to be quite fast. If feel confused, spend extra time to understand as it is easy to fall behind with doubts and concerns.

• Page: http://www.math.duke.edu/~leili/, then click ‘Teaching-Math 212’

• Sakai: Helpful materials for all Math 212 students on the shared site and grades on our own site.

• Office hours: 11:00am-12:00pm, Tuesday, 4:00-5:00pm Wed or by appointment. Physics 222. Help room: encourage you to go there. Academic Resource Center

• Homework assignments: finalized every Friday and collected every Monday

• There are some in-class or taken-home quizzes, to be announced.

• HW: 10%; Quiz: 5%; Mid-terms: 3*15%; Final 40%. Final exam: the four chapters will contribute roughly 20/20/20/40

12.1-Vectors in the plane

Def.

• Scalar: a single real number (pressure, speed)

• Vector: Both magnitude and direction (force, velocity). A directed line segment is used to represent a vector. Can move the vectors freely as long as the directed line segment carries the same magnitude and direction.
  
  − \( \vec{QR} \) is a vector going from \( Q \) to \( R \).
  
  − For a point \( P \) in space, we make a vector starting from the origin \( O \) and ending at \( P \). The vector \( \vec{OP} \) describes the position of \( P \), and is called the position vector.
When typing vectors denoted by a single letter, we sometimes use bold type \textbf{a} or a letter with an arrow \textbf{a}. When we write by hand, we should use \textbf{a}.

Vectors and ordered pairs of real numbers

1. In a Cartesian plane:
   - \textbf{i}: a vector with magnitude 1 along \textit{x}-axis;
   - \textbf{j}: a vector with magnitude 1 along \textit{y}-axis.

   Any vector \textbf{v} in the plane is expressed as \textbf{v} = ai + bj for some real numbers \(a, b\). Then, the ordered pair \((a, b)\) represents uniquely a vector.

   Since \textbf{i} = 1 \cdot \textbf{i} + 0 \cdot \textbf{j}, then \textbf{i} = \langle 1, 0 \rangle. Similarly, \textbf{j} = \langle 0, 1 \rangle.

   Given \(Q(a_1, b_1), R(a_2, b_2), \overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = \langle a_2 - a_1, b_2 - b_1 \rangle\).

2. \(\textbf{v} = \langle a, b \rangle\). **Components:** \(a, b\); length or magnitude: \(\sqrt{a^2 + b^2}\) (Pythagorean theorem).

   Vectors in 2D plane: 2 components; Vectors in 3D space: 3 components.

   **Zero vector** is a vector with length zero, denoted as \(\textbf{0}\). The direction is arbitrary.

3. **Unit vector:** A vector with length 1. If \(\textbf{a} \neq \textbf{0}\), \(\textbf{u} = \frac{1}{|\textbf{a}|} \textbf{a}\) is a unit vector that shares the direction with \textbf{a}. We use \textbf{u} to represent the direction of \textbf{a}. Sometimes denote it as \(\hat{\textbf{a}}\).

   **Example:** Consider \(A(1, -2)\) and \(B(-1, -2 + 2\sqrt{3})\). Let \(\textbf{v} = \overrightarrow{AB}\). Find the magnitude and direction.

   **Solution.** \(v = |\textbf{v}| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{16} = 4\).

   \(\hat{\textbf{v}} = \frac{1}{|\textbf{v}|} \textbf{v} = \frac{1}{2} \langle -1, \sqrt{3} \rangle.\)

Basic operations

1. \(\textbf{u} = \langle u_1, u_2 \rangle, \textbf{v} = \langle v_1, v_2 \rangle\). \(\textbf{u} = \textbf{v}\) iff \(u_1 = v_1, u_2 = v_2\).

2. Addition and Scalar multiplication are performed componentwise.
3. Geometric meanings:

Addition: Triangle law or parallelogram law. Triangle rule indicates:
\[ \vec{A}_1 \vec{A}_2 + \vec{A}_2 \vec{A}_3 + \ldots + \vec{A}_{n-1} \vec{A}_n = \vec{A}_1 \vec{A}_n \]

Scalar multiplication: Scale the length. \( c > 0 \): the same direction; \( c < 0 \): opposite direction.

One useful identity:
\[ |cv| = |c||v| \]

Proof. \( |c\langle v_1, v_2 \rangle| = |\langle cv_1, cv_2 \rangle| = \sqrt{c^2 v_1^2 + c^2 v_2^2} = |c|\sqrt{v_1^2 + v_2^2} \)

More examples

Example Let \( u = 2i + 3j, v = -2i + j \). Find \( |8u - 4v| \).

Solution. \( |8u - 4v| = 4|2u - v| \). \( u = \langle 2, 3 \rangle, v = \langle -2, 1 \rangle \). \( 2u - v = \langle 4, 6 \rangle - \langle -2, 1 \rangle = \langle 6, 5 \rangle \). Hence, \( |2u - v| = \sqrt{6^2 + 5^2} = \sqrt{61} \). The final answer is \( 4\sqrt{61} \).

Read the example about relative velocity in book. We omit it here.

Example: An application In \( \triangle ABC \), \( M, N \) are the midpoints of \( AB \) and \( AC \). Show that \( |MN| = \frac{1}{2}|BC| \).