Quiz 6

Let \( \mathbf{F} = xy^2 \mathbf{i} + x^2y \mathbf{j} \). \( R \) is the region \( x^2 + y^2 \leq 4, x \geq 0 \). Let \( C \) be the boundary of the region oriented counterclockwisely. Compute the circulation \( \oint_C \mathbf{F} \cdot d\mathbf{r} \) and the outer flux \( \iint_C \mathbf{F} \cdot \mathbf{n}ds \) using Green’s theorem.

Applying Green’s theorem

\[
\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C xy^2 \, dx + x^2y \, dy = \iint_R (2xy - 2xy) \, dA = 0.
\]

For the circulation, if you notice \( Q_x = P_y \), then it is conservative. The integral of a conservative field on a closed loop must be zero.

For the flux:

\[
\oint_C \mathbf{F} \cdot \mathbf{n}ds = \iint_R \nabla \cdot \mathbf{F} \, dA = \iint_R (x^2 + y^2) \, dA = \int_{-\pi/2}^{\pi/2} \int_0^2 r^2 r \, dr \, d\theta = 4\pi.
\]

Here, \( \text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F} \), not \( \nabla \mathbf{F} \) or \( \mathbf{F} \cdot \nabla \). The region if half of the disk and that is why we use polar.