Quiz 1

Find a scalar equation for the plane passing through $P(1, 0, -1)$ and perpendicular with the line of intersection of planes $x + y + z = 5$ and $3x - y = 4$.

We provide three solutions

Solution. By the description, a normal vector of the new plane is parallel with the line of the intersection. The direction of the line of intersection, however, is parallel with the cross product of the normal vectors of the two old planes. $n_1 = (1, 1, 1)$ and $n_2 = (3, -1, 0)$. Hence,

$$n = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 3 & -1 & 0 \end{vmatrix} = (1, 3, -4).$$

Hence, the new plane is given by

$$1(x - 1) + 3(y - 0) + (-4)(x - (-1)) = 0 \Rightarrow x + 3y - 4z = 5.$$

□

Solution. By the description, a normal vector of the new plane is parallel with the line of the intersection. We solve for the line of intersection directly. We regard $z$ as the parameter and solve $x, y$ in terms of the parameter $z$:

$$x + y = 5 - z$$
$$3x - y = 4$$

We find $x = (9 - z)/4$ and $y = 3x - 4 = 11/4 - 3z/4$. Hence, the position vector of the line is given by

$$r = (x, y, z) = (9/4 - z/4, 11/4 - 3z/4, z).$$

The direction can be read out to be $v = (-1/4, -3/4, 1)$. The the new plane is given by

$$-\frac{1}{4}(x - 1) - \frac{3}{4}(y - 0) + (x - (-1)) = 0 \Rightarrow x + 3y - 4z = 5.$$

□
Solution. By the description, a normal vector of the new plane is parallel with the line of the intersection. To find the direction of the line, we find two points on the intersection. Let $x = 0$, we see $y = -4$ and $z = 9$. Hence, one point is $(0, -4, 9)$. A second point can be found by letting $x = 1$: $y = 3 - 4 = -1$ and $z = 5$. Hence, a second point is $(1, -1, 5)$. A vector that is parallel with the line of intersection is given by

$$v = \langle 1, -1, 5 \rangle - \langle 0, -4, 9 \rangle = \langle 1, 3, -4 \rangle.$$ 

Hence, the new plane is given by

$$1(x - 1) + 3(y - 0) + (-4)(x - (-1)) = 0 \Rightarrow x + 3y - 4z = 5.$$ 

□