Practice problems

1. The cone \( z^2 = x^2 + y^2 \) and the plane \( 2x + 3y + 4z + 2 = 0 \) intersect in an ellipse. Find the tangent line of the ellipse at \( P(3, 4, -5) \). Find the plane that is normal to the ellipse at \( P(3, 4, -5) \). (This is essentially your homework problem)

2. Find a plane that is tangent to the paraboloid \( z = 2x^2 + 3y^2 \) and is parallel with \( 4x - 3y - z = 10 \). What is the distance between the plane you find to the plane \( 4x - 3y - z = 10 \)? (This is also essentially your homework problem.)

3. \( F(x, y, z) = xyz + x^2 - 2y^2 + z^3 \). Find the tangent plane of the level set \( F = 14 \) at \( P(5, -2, 3) \). Find \( u \) such that \( D_u F \) is the largest at \( P(5, -2, 3) \).

4. \( z = 2x^4 - 8xy + 2y^4 \). Is there a highest point on the graph? If yes, find it. Is there a lowest point on the graph? If yes, find it.

5. (a). Let \( f = 3x^2 + 4y^2 + z^2 \) and \( g = 2x + 3y + z = 1 \). Use Lagrange multiplier to find the extrema of \( f \) on \( g = 1 \). Is this a max or a min?

(b). Let \( f = 2x + 3y + z \) and \( g = 3x^2 + 4y^2 + z^2 = 1 \). Do the same questions as in (a).

6. Consider that we want to make a box with 6 faces. We want the volume to be 250 in\(^3\). The cost for the top and bottom material is 4 c/in\(^2\) and the cost for side material is 2 c/in\(^2\). What are the dimensions that minimize the cost?

7. Find the points on \( xy - z^2 + 1 = 0 \) that are closest to the origin.

8. Find the highest and lowest points on the intersection of \( x^2 + y^2 = 1 \) and \( 2x + 2y + z = 5 \).

9. Find all critical points of \( f(x, y) = x^3 + y^3 + 3xy \) and classify them.

10. Find all critical points of \( f(x, y) = 6xy^2 - 2x^3 - 3y^4 \) and classify them. For \((0, 0)\), the test fails. Check the behavior near it and convince yourself that it is a saddle point.

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1. Evaluate the double or iterated integrals:
\[ \int_R \sqrt{x^3 + 1} dA \text{ where } R = \{(x, y) : 0 \leq y \leq 1, \sqrt{y} \leq x \leq 1\}. \]

\[ \int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2) dx dy \]

\[ \int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx \]

2. Compute the volume of the solid bounded by \( y = -1, y = x + 1, y = 1 - x, z = x^2 + y^2 \) and the \( xy \) plane.

3. Evaluate the volume bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = y + 2 \).

4. Evaluate the volume bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = y \).

5. Write the following integral in the order \( dzdxdy \) and \( dxdydz \)
   \[ \int_{\sqrt{3}/2}^{\sqrt{3}/2} \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \int_{1-x^2-y^2}^{1-x^2-y^2} f(x, y, z) dz dy dx. \]

6. Let \( T \) be the region bounded by \( y = x^2, x = y^2, z = 0 \) and \( z = x + y \). Find the triple integral \( \iiint_T xydV \).

7. Let \( R \) be the region bounded by \( 2x + y = 3, y - 2x = 2, 2x + y = 1, 2x - y = -4 \). Compute the double integral
   \[ \iint_R (16x^2 - 4y^2) dxdy. \]

8. Compute the area bounded by \( xy = 1, y = 3/x, y = 2x^2, y = 4x^2 \).

9. Consider that \( T \) is given by \( x^2 + y^2/4 + z^2/9 \leq 1 \). Evaluate the integral \( \iiint_T x^2 dV \).

10. Find the volume under \( f(x, y) = x \) and above the region \( R = \{(x, y) : (x - 1)^2 + y^2 \leq 1, x^2 + (y - 1)^2 \leq 1\} \).

11. Consider the the solid bounded by \( \phi = \pi/6 \) and \( \rho = 2 \cos \phi \). Suppose the density is given by \( \delta = x^2 + y^2 \). Find the centroid and moment of inertia about \( z \) axis.

12. Evaluate \( \iiint_T xydV \) where \( T \) is the region bounded by \( x^2 + y^2 - 2x = 0 \) and \( x^2 + y^2 + z^2 = 4 \). What is the volume of this region?

13. Set up the integral for the mass of the region contained in the sphere \( x^2 + y^2 + (z - a)^2 = a^2 \) but below \( z = r \) with unit density.