Inclusive Teaching Practices in Mathematics

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Abstract

We summarize some "best" practices for inclusive undergraduate mathematics instruction. We intend this document as a quick reference guide for the busy instructor. Each section consists of a bulleted list of practices, with longer sub-bullets providing elaboration, examples, and additional sources. Practices are organized both by their position relative to the course and by their relevance to various student identities. We assembled these practices from a variety of research articles, university teaching center resources, and existing longer guides.

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1 Motivation

• "Establishing a supportive and encouraging learning community is crucial to attracting new majors and to nurturing them as mathematics learners. As a first principle, a

department must show that it respects and values students, [recognizes] the needs of different student populations, and [works] to ensure that all feel welcome." [SB15]

- Inclusive classroom and department practices support retention of underrepresented groups in STEM [SH19, EKR14].
- Right now, members of underrepresented groups choose to leave mathematics at every career stage. For example:

Percentage of women in 2015 among US mathematics	
undergrads	42%
undergrads at PhD-granting insitutions	32%
first-year graduate students	33%
full-time graduate students	30%
faculty	30%
faculty at PhD-granting institution	24%

Table 1: Data from [BKM18] tables E.1.A and F.5, and [GBR19] table GS.2.

• Other underrepresented groups also experience poor attrition in mathematics. At the faculty level, one has:

	Percentage of Full-time Faculty						
	Asian	Black, not Hispanic	Mexican American/ Puerto Rican/ other Hispanic	White, not Hispanic	AIAN or NHPI ¹	Unknown	
	%	%	%	%	%	%	
PhD Mathematics Departments							
All full-time men	15	1	3	55	0	2	
All full-time women	5	0	1	16	0	1	
MA Mathematics Departments							
All full-time men	11	2	3	46	0	2	
All full-time women	6	1	1	26	0	1	
BA Mathematics Departments							
All full-time men	6	2	1	53	0	2	
All full-time women	4	1	1	30	0	1	

Figure 1: Table F.5 of [BKM18]. The acronym AIAN or NHPI means American Indian, Alaskan Native, Native Hawaiian, or other Pacific. An entry of 0 indicates less than 0.5%.

For comparison, the US Census Bureau estimated in 2015 that 13% of Americans were Black and 1% were American Indian, Alaskan Native, Native Hawaiian, or other Pacific [Bur15]. Research literature increasingly studies the effects of "intersecting" marginalized identities – note the stark differences between men and women within the same racial category above.

2 General Recommendations

2.1 Department-level recommendations

- Advertise department activities in class and support students in forming study groups.
 - "Department-sponsored activities like seminars and guest speakers, contests, picnics, and field trips help build community. Study groups create bonds between students while helping them learn." [CUP15]
- Invite diverse speakers to contribute to department and/or class activities.
 - "It is equally important to be sure that invited speakers are diverse in gender, culture, race, and undergraduate major. Students respond well to such visitors because they want to see how their education will be integrated into their careers. They need to be able to see the possibilities for themselves in their life after college. They also benefit from seeing that gender and racial stereotypes can be overcome." [CUP15]
- Consider whether course and/or departmental policies reinforce the *hidden curriculum*.
 - "Hidden curriculum" describes "what is implicit and embedded in educational experiences in contrast with the formal statements about curricula and the surface features of educational interaction." [SM98]
 - Adhere to clearly stated course policies so that students familiar with negotiating these are not given an advantage over those who interpret the policies as rigid and nonnegotiable.
 - Adopt strategies to uncover the hidden curriculum at the departmental and course levels to raise awareness and enact change [SB18].
- Support departmental structures that reward thoughtful pedagogy and organizing community activities.
 - "A critical mass of department faculty must be committed to and involved in the process of creating and sustaining community... Forming a vibrant student community begins with the department's conscious commitment to this effort."
 [CUP15]
 - "Align the faculty reward structure with educational goals. Faculty who want to experiment with pedagogy often complain that such work is both time-consuming and unrewarded in the promotion, tenure, and merit processes. Failing to align rewards with department needs for renewal and reform leads to stagnation." [CUP15]

2.2 Course-level recommendations

- Encourage students to use a variety of learning methods and mediums (e.g. auditory, visual, and written).
 - A common misconception is that a person best processes information when it matches their preferred "cognitive style" (e.g. auditory). Students benefit from exposure to a variety of styles, especially when they are taught how to select the most appropriate style for a given situation. [SB15]
- Encourage students to develop and value a growth mindset instead of a fixed mindset.¹
 - Specified to mathematics, a fixed mindset implies that students believe that their talent in mathematics is fixed. That is, one is either good at mathematics or one is not. [ABE⁺18]
 - A growth mindset in mathematics implies that students view challenge and struggle as avenues to shape and expand their mathematical understanding and that mathematical talent is malleable. [ABE⁺18]
- Adopt anti-deficit teaching practices: encourage student questions and celebrate partial understanding as a step toward full understanding.
 - "Instructors must deliberately adopt an anti-deficit perspective on students and their knowledge in order to recognize that all students have the ability to contribute in the classroom. Misconceptions and errors in student thinking are a natural part of learning. The fixation on remediation is deficit-oriented, undermines student progress, and hinders the development of mathematical identity. The value of students' ideas should not be solely based on proximity to the norm." [ABE⁺18]

2.3 Course-design recommendations: "universal design"

- Incorporate *universal design* practices into your course design.
 - "Universal design: Also known as 'inclusive design' and 'design for all', this is an approach to the design of products, places, policies and services that can meet the needs of as many people as possible throughout their lifetime, regardless of age, ability, or situation." [Har20]
 - "Accessibility: The 'ability to access' the functionality of a system or entity and gain the related benefits. The degree to which a product, service, or environment is accessible by as many people as possible." [Har20]
 - "Universal design ensures that an environment can be accessed, understood, and used to the greatest extent possible by all people." [Har20]
- Examples of universal design practices:

¹Note that there is some ambiguity in the literature surrounding growth mindsets [MB23]

- Example 1: the on-campus testing center not only provides a space for students with disabilities to access testing accommodations, but also allows all students to take makeup exams (after, say, a sickness or family emergency).
- Example 2: recording and posting lectures not only supports students who have a disability that limits their note-taking capacity, but also provides all students with the ability to access lectures for a day they must miss.
- Note that many practices in Section 3 (identity-specific practices) exemplify universal design principles. Inclusivity is for everyone.
 - Example 1: the communication strategies in Section 3.1 benefit everyone, not just students with English as a second language.
 - Example 2: the use of diverse names, mentioned in Section 3.2, can also support the inclusivity of people by gender, country of origin, use of a "nickname", etc.

2.4 Syllabus and the first week

- Use the syllabus guidelines, templates, or language menus provided by your college/university, or those of others. Use student-centered language.
 - For example, Duke's "Learning Innovation and Lifetime Education" maintains a student-centered syllabus template at https://lile.duke.edu/resources/templates/.
- Include learning outcomes, campus resources, student conduct expectations, the grading structure of the course, and clearly stated course policies. Preemptively connecting students with resources not only helps students succeed, but can also reduce student emails to instructors.
 - Consider renaming "office hours" to "student help hours". The phrase "student help" more clearly describes their purpose, and can feel less intimidating to students. Either way, clearly communicate with students the purpose/structure of office hours: emphasize that students may come and go as they please, and that office hours exist to support them!
- Review the syllabus in class on the first day of class, and communicate that serves as an agreement between the instructor and students.
 - "Setting the classroom norms and engagement expectations to incorporate studentcentered learning on the first day of class is important... For example, taking time to build a classroom community allows students to form and broaden their support networks outside of class which can include other students, instructors, or other campus resources. Another benefit of such networks is that they play an important role in the success of members of marginalized communities" [ABE⁺18].
- Example day 1 and day 2 structure:

- Day 1: distribute note cards on which students can write their name and pronouns for the class. Share three interesting facts about yourself. Then have students write 3 interesting facts about themselves and pair up to share. Finish with a guided math task, and an assignment that encourages students to read the syllabus and check out resources.
- Day 2: ask students to have name cards (or instructor can collect and distribute).
 When answering students questions, use students' names.
- Share as much as you're comfortable about your own motivation for teaching the course, along with what you've learned in teaching it previously [SB18].

2.5 In the classroom

- "Prioritize: If everything is important, nothing is important. Let students know what is most essential to read, do, remember, revise, etc." [IWB⁺]
 - "Students have difficulty identifying the most important aspects of what they are seeing and hearing [in lecture]. Either they try to record everything, creating notes that are of little use, or they focus on what they imagine to be important, the template solutions. The same is true when students are 'studying'. They focus on what they know how to do and what they expect will be important on the examinations, learning template solutions." [CUP15]
- Wait at least seven seconds before answering a question posed to the class.
 - "On average, instructors wait less than 1.5 seconds before they either answer their own question or ask a follow-up question... Research shows that an average wait time greater than three seconds is a threshold for changing instructor-student discourse" [ABE⁺18]
 - "Short wait times lower the cognitive demand of tasks and discourage students' deep engagement in mathematics. A short wait time also communicates to students that a response is not actually necessary or that they are not expected to answer questions." [ABE⁺18]
 - Consider repeating the question after a long wait time, or asking students if they need you to repeat it.
 - Remind students why you are waiting and the benefits for them when you wait.
 - If you reach 10 or more seconds with no response, consider making use of Think-Pair-Share.
- "[Mathematics] instructors should present key ideas and concepts from a variety of perspectives, employ a broad range of examples and applications to motivate and illustrate the material, promote awareness of connections to other subjects, and introduce contemporary topics and their applications." [SB15]
- Incorporate group work on material with a "low floor" and a "high ceiling".

- Since the instructor cannot talk to all groups simultaneously, students need to have an entry point to productively engage with material (low floor). At the same time, tasks should be complex enough to spark interest and require meaningful thought to answer (high ceiling). [ABE⁺18]
- If grouping students based on performance, it is important to place low-performing students with medium-performing students and medium-performing students with high-performing students. [ABE⁺18]
- Example from Abstract Algebra: [ABE⁺18]
 - * Calculate 0, 1, and -1 in the ring $\mathbb{Z}/4$.
 - * Calculate 0, 1, and -1 in the rings $M_2(\mathbb{R})$ and $M_2(\mathbb{C})$.
 - * In the ring $M_2(\mathbb{R})$, compute the element $3 \cdot 1$ (encourage multiple approaches, e.g. scalar multiplication or repeated addition)
 - * Given a Cayley Table for some finite group, calculate 0 and 1 in this ring; calculate -1 in this ring; calculate $3 \cdot 1$ in this ring.
- When forming groups for labs/discussions, try to ensure that groups with women have at least two women in them. [WM11, DSH15]
 - At the same time, if the class has exactly 3 women (or exactly 3 students of a particular identity), do not automatically place those students in a group together. Doing so can make the students feel "othered."
 - Alternatively, random groups can be extremely beneficial to students' learning as long as the randomness is visible to students – for example, by drawing cards from a deck. [Lil20] Regularly-changed, random groups also mitigate identitybased "othering".

2.6 Communication with students

- "Predictability: Establish and maintain clear expectations, demonstrating dependability, and giving students advance notice of changes. Build off your college/university policies that are clear and implement them consistently. Avoid surprises and disappointment." [IWB⁺]
- Below are some selected communication tips: [U.S94]
 - organize material with short sections and short sentences;
 - use lists and tables to simplify complex material; and
 - minimize the number of subordinate levels in lists.

3 Identity-specific Inclusivity Practices

3.1 English as a second language inclusivity

• Speak (in lecture and office hours) and write (in written notes, the course syllabus, and written communication) plainly, using the simplest wording possible [U.S94]. Use

short sentences and the simplest tense possible. Use concrete, familiar words. Some examples:

- "come to office hours" or "visit office hours" instead of "stop by office hours"
- "join this discussion" instead of "jump in here"
- "write your solution" instead of "write up your solution"
- Consider differences in mathematical notation [Lib10]. While it's reasonable to use a standard notation in your class, know that some students will be adjusting from the notation they've seen before your class. Some examples (from [Lib10]):
 - decimal numbers (the decimal separator can be indicated with a period or a comma)
 - names of functions ("sin" as shorthand for the sine function is an English language abbreviation; a student whose first language is Spanish may be inclined to use "sen")

3.2 Race inclusivity

- Anti-deficit teaching and inquiry-based learning practices (see the "general" section above) improve outcomes for students of color [ABE⁺18]
- Ask students to relate mathematical concepts to real-life scenarios (as analogies, not direct applications) to strengthen understanding and counter a deficit narrative [AZ20].
 - For example, consider combinations of baking ingredients sugar, salt, flour, etc.
 and the various recipes they can form. The ingredients form a "basis" which "span" the "space" of baked goods (see Table 5 of [AZ20] for more examples made by students).
- Recognize and counter *stereotype threat*: "respect each of your students as individual learners and encourage a growth mindset in the classroom. This means normalizing mistakes and failures, emphasizing the value of challenge, and offering students a variety of ways to demonstrate their learning." [Har24]
 - Stereotype threat refers to "the anxiety individuals from stigmatized groups have that their behaviour might confirm – to others or even to themselves – the negative stereotypes imposed upon their group". [BF12]
- Use names (in word problems and examples) from a variety of origins/cultures.

3.3 Gender inclusivity

• If the name a student shares with you differs from their legal name or paperwork, do not label it a "nickname" unless a student does so first. [IWB⁺]

- Introduce yourself with pronouns: this normalizes sharing pronouns in class, which reduces a burden that often falls on trans students.
- Give students a space to share their pronouns if they wish, but never require students to share their pronouns. Students may have the option to share pronouns through their registrar profile; review this in your class roster at the beginning of the semester.
- If you accidentally misgender a student, apologize, correct your language, and move on. Over apologizing and dwelling on the mistake can cause more harm and forces the student to put more effort into addressing your emotional response. [IWB⁺]
- Be mindful of mathematics that either assumes or reinforces a traditional male/female gender binary. [IWB⁺]
 - Avoid statistics examples that pathologize men or women based on physical characteristics (e.g. instead of height, use blood pressure). Consider a discussion of bimodal distributions if applicable.
- Find opportunities to normalize and include they/them pronouns or neopronouns (e.g. by discussing the contributions of queer mathematicians, or pronoun choices in word problems). [IWB⁺]

3.4 Sexuality inclusivity

- Hall's Marriage Theorem: avoid the traditional, heteronormative marriage motivation, and consider referring to the theorem as "Hall's Theorem".
 - For an alternative, pair medical students with residency programs, a group of friends with gifts, or variations thereof.
- More generally, avoid language and examples that assume heteros exuality as a default. $[\rm IWB^+]$

3.5 Disability inclusivity

- Students do not need to disclose specific details to their instructors about their disabilities to receive accommodations, and it infringes on the student's privacy to inquire.
 - "Students with disabilities... have to navigate learning environments differently from other students. We recognize that we are not experts on the particular needs of a student. For example, we cannot treat all students in a wheelchair the same way because they will likely have different needs. We rely on the assistance from both the student and the office for disability services on campus to understand the student's particular needs." [ABE⁺18]
- Avoid words that assume neurotypical or able-bodied as a default.
 - e.g. avoid "normal" (except in a technical sense, as in the normal distribution), "(physically) fit", "insane/crazy", etc.

3.6 Income/class inclusivity

- Think carefully about what resources students must access for the class.
 - Reduced access to printing, a laptop, a textbook, transportation, etc. can be limiting.
 - Connect students with financial aid resources (e.g. in the syllabus).
- When writing/choosing word problems involving money, consider student familiarity with financial language and the sizes of the amounts involved.
- "Do not assume that all students have conventional housing circumstances, family support, financial stability, or access to necessary medical care." [IWB⁺]

4 Additional Resources

- Creating a Syllabus: https://www.chronicle.com/article/how-to-create-a-syllabus
- Bias Free language: https://apastyle.apa.org/style-grammar-guidelines/bias-free-language
- A 2012 report on producing more STEM graduates in the U.S.: https://files.eric.ed.gov/fulltext/ED541511.pdf
- MAA Instructional Practices Guide: http://sigmaa.maa.org/rume/RUME25_Proceedings.pdf
- The Path to College Calculus: The Impact of High School Mathematics Coursework: https://pubs.nctm.org/view/journals/jrme/49/3/article-p292.xml
- Tips for culturally responsive teaching: www.edutopia.org/article/5-tips-culturally-responsive-teaching-math-classroom
- Identity-specific resources:
 - Hispanic Heritage Month math resources: https://sabes.org/content/latinxhispanic-heritage-math-resources
 - Center for Math Education of Latines: https://cemela.math.arizona.edu/home
 - Native American Heritage Month math resources: https://sabes.org/content/native-american-heritage-math-resources
 - LGBTQ+ math resources: https://sabes.org/content/lgbtqia-math-resources

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