Course syllabus for Math 231br: Advanced algebraic topology

Spring 2013

Course description: many objects of mathematical interest can be expressed as the set of fixed points of a group acting on another object. For example, if $f_1, f_2, \ldots, f_n$ are polynomial equations in m variables $x_1, \ldots, x_m$ with coefficients in $\mathbb{Q}$, the solutions $(x_1, \ldots, x_m) \in \mathbb{Q}^m$ are the fixed points of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acting on the solutions in $\overline{\mathbb{Q}}^m$. The fixed points of a group action are an example of a limit. Algebraic topology provides a related notion called a *homotopy limit*, which has better properties and is easier to calculate. There is a beautiful result, called the Sullivan conjecture, which gives an equivalence between fixed points and homotopy fixed points in a certain context. Sullivan's conjecture was proven independently by Haynes Miller, Gunnar Carlsson, and Jean Lannes. This course will be a second semester algebraic topology course which will continue the development of the standard tools of algebraic topology introduced in Math 231a while working towards a proof of Sullivan's conjecture.

Time and place: MWF 2. Science Center 110.

e-mail and office: kwickelg@math.harvard.edu. Science Center 239.

Office hours: Monday 3-4 or by appointment.

Course assistant: Akhil Mathew, amathew@college.harvard.edu.

Section: Mondays from 8-9pm in SC 310. Office hours will precede or follow section.

Problem sets due on Tuesdays.

Topics (not necessarily in this order):

- Cohomology operations

- Steenrod algebra; modules over the Steenrod algebra
- Spectral sequences; unstable Adams spectral sequence
- Homotopy limits and colimits
- Proof of Sullivan's conjecture

Text: Robert Mosher and Martin Tangora *Cohomology operations and applications in homotopy theory* Harper & Row Publishers, New York, 1968, p. x+214.

Other references:

- A. Bousfield and D. Kan, *Homotopy limits, completions and localizations*, Lecture Notes in Mathematics, Vol. 304, Springer-Verlag, Berlin, 1972, v+348.
- Gunnar Carlsson, *G. B. Segal's Burnside ring conjecture for* $(\mathbf{Z}/2)^k$, Topology, **22**, 1983, no. 1, 83–103.
- Gunnar Carlsson, *Equivariant stable homotopy and Sullivan's conjecture*, Invent. Math. **103** (1991), no. 3, 497–525.
- William Dwyer, Haynes Miller, and Joseph Neisendorfer, *Fibrewise completion and unstable Adams spectral sequences*, Israel J. Math. **66** (1989), no. 1-3, 160–178.
- Jean Lannes, em Sur les espaces fonctionnels dont la source est le classifiant d'un p-groupe abélien élémentaire, with an appendix by Michel Zisman, Inst. Hautes Études Sci. Publ. Math., **75**, 1992, 135–244.
- Haynes Miller, *The Sullivan conjecture on maps from classifying spaces*, Ann. of Math. (2) **120** (1984), no. 1, 39–87.

  There are two minor corrections here:
- Haynes Miller, *Correction to: "The Sullivan conjecture on maps from classifying spaces"* , Ann. of Math. (2), **121**, 1985, no. 3, 605–609.

Prerequisites: Math 231a or the equivalent. You do not need to read french to take this course. Course notes will be supplied on the material from Lannes's article.

Homework, and grading: Grades will be based on problem sets, and a 4-10 page paper to be completed by the end of reading period. There will be a problem set once a week while we cover the first four topics, and then once every two weeks. I'll provide a list of potential topics for the final paper half way through the course, but you are also welcome to chose another topic which interests you. Feel free to work together on problem sets. Hand in your own write-up. The names of your collaborators should appear on your work.