Monday Jury duty

Holiday if you know Tot

otherwise: (co)Simplicial sets E. Riehl

L15 3/7/13

Nerves, EG, group coh, $K: X^G \to H^1(G, \pi_1)$, $\downarrow$ X^{hG} $\nearrow$

Spectral sequence associated to tower of fib

HW correction (J. Shah): doesn't depend parity corrected website PS4 #7

Recall: build space by gluing

0 - simplices

1 - simplices

$\vdots$

n - simplices

$\vdots$

This data is a functor

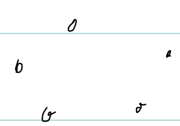
$n \longmapsto X_n$ = set of n simplices

associated space "geometric realization"

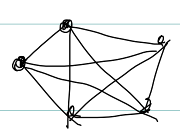
$\Delta = \{\underline{0}, \underline{1}, \underline{2}, \ldots, \underline{n}, \ldots\}$

$\underline{n} = \{0, 1, \ldots, n\}$

EG contractible space w/ free G-action G discrete group



0 simplices : G



1 simplices: $G \times G$

$\vdots$

n simplices: $\overset{n+1}{\times} G$

Systematically: Given cat C, $NC \in sSet = Fun(\Delta, Set)$

$NC_n = \{$ n - composable morphisms in $C \} = Fun(\underline{n}, C)$

$\underline{n}$ is viewed as cat

EG is geometric realization of

cat w/ ob: G

unique morphism b/w any two objects

$BG = EG/G$

<u>Group coh</u>

<u>Def 1</u>: $H^*_{group}(G, M) = H^*(G, M)$

$$:= H^*(BG, M)$$

local coefs

- simplicial chains on EG

$$\begin{array}{cc} \downarrow & \\ \mathbb{Z}[G \times G] & \\ \downarrow & | \\ \mathbb{Z}[G] & 0 \end{array}$$

is a resolution of $\mathbb{Z}$ since EG contractible

<u>Def 2:</u> $H^*_{\text{Group}}(G, M) = \operatorname{Ext}^*_{\mathbb{Z}[G]\text{-mod}}(\mathbb{Z}, M)$

<u>ex:</u> $H^1(G, M) = \dfrac{\{\sigma : G \to M \mid \sigma(gh) = \sigma(g) + g\sigma(h)\}}{\{\sigma(g) = m - gm\}}$

This even works for non-ab M

becomes give

S'pose $G \circlearrowright X$ and $b \in X^G$

Say relation to Kummer map from Selmer group and $\mathbb{G}_m$

· Give construction & ext'n of construction from Madison talk up to $\operatorname{Map}(EG, X)^{G}$ filtered $\rightsquigarrow$ spectral sequence

(extended htpy)
Spectral sequence associated to a tower of fibrations

reference: Bousfield, Kan "htpy limits, completions, & localizations"

Tower of fibrations

$$\begin{array}{c}\downarrow \\ X_n \leftarrow F_n = \mathrm{Fib}(p) \\ \downarrow p \\ X_{n-1} \\ \downarrow \\ \vdots \\ \\ X_0 \\ \downarrow \\ * = X_{-1}\end{array}$$

$X = \varprojlim X_n$

Q: Which elts of $\pi_* F_n$ represent elts of $\pi_* X$ in the sense that

$$a \in \pi_* F_n$$

has image in $\operatorname{im}\left(\pi_* X \longrightarrow \pi_* X_n\right)$?

<u>Rmk:</u> $a \in \operatorname{Ker}\left(\pi_* X_n \to \pi_* X_{n-1}\right)$

So if a reps elt of $\pi_* X$, this elt is automatically in $\operatorname{Ker}\left(\pi_* X \longrightarrow \pi_* X_{n-1}\right)$

These kernels will filter $\pi_* X$

For now: pretend all htpy groups abelian

$$\begin{tikzcd}
\pi_{*-1} F_{n+3} & \pi_* X_{n+2} \arrow[l] \arrow[d] & \\
\pi_{*-1} F_{n+2} & \pi_* X_{n+1} \arrow[l] \arrow[d] & \\
\pi_{*-1} F_{n+1} & \pi_* X_n \arrow[l] & \pi_* F_n \arrow[l]
\end{tikzcd}$$

$$\cdots \longrightarrow \pi_* F_n \longrightarrow \pi_* X_n \longrightarrow \pi_* X_{n-1} \longrightarrow \pi_{*-1} F_n \longrightarrow \cdots$$

Let $\pi_* X_n^{(r)} = \operatorname{im}\left(\pi_* X_{n+r} \longrightarrow \pi_* X_n\right)$

$$\cdots \longrightarrow \pi_* X_n^{(r)} \longrightarrow \pi_* X_{n-1}^{(r)} \longrightarrow \pi_{*-1} F_{n-1+r}^{(r)} \longrightarrow \cdots$$

$$\pi_* F_n^{(r)} = \frac{\ker\left(\pi_* F_n \to \dfrac{\pi_* X_n}{\pi_* X_n^{(r)}}\right)}{\ker\left(\pi_{*+1} X_{n-1} \to \pi_{*+1} X_{n-1-r}\right)}$$

The $\pi_* F_n^{(r)}$ form a spectral sequence

$$E_r^{-s,t} = \pi_{t-s} F_s^{(r-1)} \qquad s \geq 0$$