MATH 690-10.01
Topology: Characteristic classes and applications

Fall 2019

Course description: Vector bundles are of fundamental importance in mathematics and its applications, but locally, they are the same if they have the same dimension. Characteristic classes record information about the global structure of vector bundles in the form of cohomology classes. They are a powerful and beautiful tool, arising in algebraic topology, algebraic geometry, arithmetic geometry, differential geometry, and mathematical physics. They can distinguish between the tangent bundles of two different smooth structures on the same topological manifold, and answer the question "how many lines meet four lines in $\mathbb{R}^3$?" This course will serve as a second course in algebraic topology, together with applications to algebraic geometry and number theory. We will use Milnor and Stasheff's classic text, and also discuss open problems.

Some references: Characteristic Classes by J.Milnor and J. Stasheff. *Vector Bundles and K-theory* by A. Hatcher.

Time and place: MW 1:25pm-2:40pm. Allen 304I. Webpage through link at `https://services.math.duke.edu/~kgw/`.

Exception for first day of class I am involved in the Women in Topology program:

`https://www.him.uni-bonn.de/events/scientific-events/single-scientific-events/women-in-topology-2019/description/`

The program consists of teams working collaboratively on a problem for about a year and a half (let me know if you're interested! It is very likely that there will be further programs). There is one in-person meeting, and unfortunately, it is the first week of class. I'm co-leading a research team, so I need to be in Bonn, Germany, August 26-30. Sorry for the inconvenience! Here's what we'll do. For the first day of class, we'll do a video conference **meeting in the conference room at Duke University**, and I'll give class from Bonn. We

will not have class Wednesday August 28, but we'll make up the time during the week of December 2-6.

Instructor contact information and office hours: Kirsten Graham Wickelgren,

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office: Physics and Math: room 025

office hours: after class and by appointment.

Prerequisites: A semester of algebraic topology is advisable, but if you've got background in algebraic geometry or Riemann surfaces instead, that may work great.

Topics:

- Vector bundles, topological, smooth, holomorphic, and algebraic

  examples: tangent bundle, cotangent bundle, what is the second derivative, anyway? (I can think of more than one answer to this question:), normal bundles, tautological bundles on Grassmannians, tautological bundles on flag manifolds, canonical bundle, building new bundles from old bundles.
- Classifying vector bundles
- Connection with locally free sheaves
- Chern classes
- Stiefel-Whitney Classes
- Orientations, Spin structures
- Cohomology of Grassmannians
- Obstruction theory
- Euler class
- Pontryagin classes

Assignments: This is likely to be a small class, so we can adjust assignments to fit your goals. My suggestion is that there be a problem set about every two weeks, and two short take-home midterms, which will substitute for problem

sets during the weeks they are given. The purpose of exams is to give us the opportunity to review material. Similarly, at the end of a topic, there will be a short, **ungraded**, in-class quiz, for your benefit. I will try to advertise important constructions and proofs, and solicit volunteers to prepare and present 5-10 minute segments of certain lectures. We will do problems together in class (you are also welcome to work on your own, if you prefer).

Grading: Grades will be based on the assignments and class participation. We will work out more specific guidelines depending on who is in the class.

Update on assignments and grading Sept 7: We will have problem sets about every two weeks, where you will choose to do about 4 problems from a list about twice as long. Hopefully, this will result in you doing the problems that will be useful to you. There will be one take-home midterm substituting for a problem set. You will have a week to do it, and it is open book. You are welcome to collaborate on the problem sets, but not on the midterm. There will be no final. You may prepare and present part or all of lecture(s) for extra credit. There will be a few uncollected, ungraded quizzes that we will go over together in class.

To grade, I'll make numeric grades by weighting the problem sets 75% and midterm 25%. You may replace your lowest problem set grade with a presentation. You may hand in corrected midterm solutions after collaborating with me or someone else and have 60% of your additional points added to your midterm. You may give a presentation for an additional 25% on the midterm, capped at 100%.

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