**Characteristic classes Problem Set 7**
Due in class Wednesday November 13.

Hand in solutions to **four** of the following problems. You are encouraged to collaborate on homework assignments. Just remember to write up your proofs separately and to acknowledge your collaborators on your work. If you're not sure what a question means, please ask.

# 1 Reading

Milnor and Stasheff Chapters 9, 10, 11.

# 2 Problems

**1.** Find the Thom complex of the nontrivial $\mathbb{R}$-line bundle on $S^1$. (Describe it in more familiar terms.)

**2**. Let $V_n = \mathcal{S}^{\oplus n} \to \mathbb{RP}^k$ denote the Whitney sum of $n$ copies of the tautological bundle. Give an equivalence $\mathrm{Th}(V_n) \simeq \mathbb{RP}^{n+k}/\mathbb{RP}^{n-1}$ between the Thom complex and stunted projective space.

**3**. Milnor–Stasheff Problem 11-A. Compute the mod 2 cohomology of $\mathbb{RP}^n$ by induction on $n$ using Poincaré duality and the cell structure of with one $r$-cell for each integer $r \geq 0$ such that the $r$-skeleton in $\mathbb{RP}^r$.

**4**. Let $T^*S^n \to S^n$ denote the cotangent bundle to the $n$-sphere. Compute the Euler class $e(T^*S^n)$ for all $n$.

**5**. Milnor–Stasheff Problem 9-A

**6**. Milnor–Stasheff Problem 9-B

**7**. Milnor–Stasheff Problem 9-C

**8**. Milnor–Stasheff Problem 11-B.

**9**. Milnor–Stasheff Problem 11-D. Show all Stiefel-Whitney numbers of a 3-manifold are 0.

**10**. Let $V$ be a rank $n$ complex vector bundle and let $L$ be a rank 1 complex vector bundle on $X$. Let $V^*$ denote the dual bundle $V^* = \mathrm{Hom}(V, \mathbb{C})$ Express $e(V^* \otimes L)$ in terms of the Chern classes of $L$ and $V$.