Midterm

Return by Monday, November 25, 2019.

Hand in solutions to **three** of the following problems. Please complete the problems without collaborators. If you're not sure what a question means, please ask. You are welcome to use *Characteristic classes* by Milnor and Stasheff *Vector Bundles and K-theory* and *Algebraic Topology* by A. Hatcher, your notes from class and your problem sets.

1 Problems

1. Let $f : \mathbb{R}\,\mathrm{Gr}_2 \times \mathbb{R}\,\mathrm{Gr}_2 \to \mathbb{R}\,\mathrm{Gr}_4$ be a continuous map such that $f^*\mathcal{S}_4 \cong \pi_1^* \det(\mathcal{S}_2) \otimes \pi_2^* \mathcal{S}_2$. Compute the induced map on cohomology

$$f^* : H^*(\mathbb{R}\,\mathrm{Gr}_4, \mathbb{Z}/2) \to H^*(\mathbb{R}\,\mathrm{Gr}_2 \times \mathbb{R}\,\mathrm{Gr}_2, \mathbb{Z}/2).$$

Here, $S_n \to \mathbb{R}\,\mathrm{Gr}_n$ denotes the tautological bundle on the real Grassmannian of $\mathbb{R}^n$-planes in $\oplus_{i=1}^{\infty}\mathbb{R}$ and π_i denotes the ith projection.

2. Let V denote the normal bundle to $\mathbb{CP}^3 \hookrightarrow \mathbb{CP}^4$. Compute the cohomology ring $H^*(\mathrm{Th}(V), \mathbb{Z})$ of the Thom space.

3. Let V denote the complex vector bundle $\mathcal{O}(1) \oplus \mathcal{O}(2) \oplus \mathcal{O}(-3) \to \mathbb{CP}^4$. Compute the cohomology ring of $H^*(\mathbb{P}(V), \mathbb{Z})$.

4. Compute the first Pontrijagin class p_1 of the underlying real vector bundle associated $\mathrm{Sym}^2 \mathcal{S}_2$, where $\mathcal{S}_2 \to \mathbb{C}\,\mathrm{Gr}_2$ denotes the tautological bundle on the Grassmannian of $\mathbb{C}^2$-planes in $\oplus_{i=1}^{\infty}\mathbb{C}$.

5. The zero locus of the polynomial $y^2 z = x(x-z)(x-2z)$ gives an embedding of the 2-Torus $X = S^1 \times S^1$ into $\mathbb{CP}^2$. Let $V \to X$ denote the normal bundle. Compute the Stiefel–Whitney classes of V.