Hand in solutions to three of the following problems. Please complete the problems without collaborators. If you’re not sure what a question means, please ask. You are welcome to use Characteristic classes by Milnor and Stasheff Vector Bundles and K-theory and Algebraic Topology by A. Hatcher, your notes from class and your problem sets.

1 Problems

1. Let \( f: \mathbb{R} \text{Gr}_2 \times \mathbb{R} \text{Gr}_2 \to \mathbb{R} \text{Gr}_4 \) be a continuous map such that \( f^* S_4 \cong \pi^*_1 \det(S_2) \otimes \pi^*_2 S_2 \). Compute the induced map on cohomology

\[ f^*: H^*(\mathbb{R} \text{Gr}_4, \mathbb{Z}/2) \to H^*(\mathbb{R} \text{Gr}_2 \times \mathbb{R} \text{Gr}_2, \mathbb{Z}/2). \]

Here, \( S_n \to \mathbb{R} \text{Gr}_n \) denotes the tautological bundle on the real Grassmannian of \( \mathbb{R}^n \)-planes in \( \bigoplus_{i=1}^{\infty} \mathbb{R} \) and \( \pi_i \) denotes the \( i \)th projection.

2. Let \( V \) denote the normal bundle to \( \mathbb{C}P^3 \hookrightarrow \mathbb{C}P^4 \). Compute the cohomology ring \( H^*(\text{Th}(V), \mathbb{Z}) \) of the Thom space.

3. Let \( V \) denote the complex vector bundle \( \mathcal{O}(1) \oplus \mathcal{O}(2) \oplus \mathcal{O}(-3) \to \mathbb{C}P^4 \). Compute the cohomology ring of \( H^*(\mathbb{F}(V), \mathbb{Z}) \).

4. Compute the first Pontrijagin class \( p_1 \) of the underlying real vector bundle associated \( \text{Sym}^2 S_2 \), where \( S_2 \to \mathbb{C} \text{Gr}_2 \) denotes the tautological bundle on the Grassmannian of \( \mathbb{C}^2 \)-planes in \( \bigoplus_{i=1}^{\infty} \mathbb{C} \).

5. The zero locus of the polynomial \( y^2 z = x(x - z)(x - 2z) \) gives an embedding of the 2-Torus \( X = S^1 \times S^1 \) into \( \mathbb{C}P^2 \). Let \( V \to X \) denote the normal bundle. Compute the Stiefel–Whitney classes of \( V \).