Exam #1

Name: __________________________

Some trig identities:

\[ \sin^2(x) + \cos^2(x) = 1 \]
\[ \tan^2(x) + 1 = \sec^2(x) \]
\[ 1 + \cot^2(x) = \csc^2(x) \]
\[ \sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y) \]
\[ \cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y) \]
\[ \cos^2(x) = \frac{1 + \cos(2x)}{2} \]
\[ \sin^2(x) = \frac{1 - \cos(2x)}{2} \]
1. Evaluate the following integrals (5 points each)

(a) \( \int \frac{(1 - 3\sqrt{x})^2}{\sqrt{x}} \, dx \)

(b) \( \int \cos(x) \sqrt{\sin(x)} \, dx \)

(c) \( \int_1^2 \frac{t+1}{\sqrt{t^2+2t}} \, dt \)

(d) \( \int_0^4 \sqrt{16 - x^2} \, dx \).
2. Find the volume that remains after a hole of radius 3 is bored through the center of a solid sphere of radius 5. (15 points)

3. (5 points) Does \( \int_{0}^{2\pi} \frac{\sin(x)}{1+x} \, dx \) have a positive or a negative value? Why? Hint: what is \( \int_{0}^{2\pi} \sin(x) \, dx \)? Why, in terms of area, is this? What effect does the \( \frac{1}{1+x} \) have on the \( \sin(x) \) from 0 to \( 2\pi \)?
4. (15 points) Use the definition of the definite integral (find the limit of Riemann sums) to evaluate \( \int_{-1}^{2} (x^2 + 1) \, dx \).

Hint: \( 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \) and \( 1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \).

5. (5 points) Evaluate \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( \sin \left( \frac{\pi}{2n} i \right) \right)^{\frac{1}{n}} \)
6. (5 points) Find \( \frac{d}{dx} \int_{-3}^{\sqrt{x}} (1 + t^2)dt \).

7. (15 points) My buoy has a half sphere on the bottom and a cone on top. The sphere has radius 3 ft and the cone has height 5 ft. My buoy filled with water and sank. I sent you all to retrieve it and drain it (you’re so nice - one point extra credit for everyone!). Once you got it to shore, how much work was required to drain it? Set up the integrals necessary to calculate the amount of work, but DO NOT EVALUATE THEM. The density of the water is 60 lb/ft\(^3\).
8. (10 points) What is the surface area created by revolving the curve \( y = \frac{2}{3}x^2 \) from \( x = 0 \) to \( x = 3 \) around the y-axis?

9. (10 points) Recall that the error \( |T E_{n}| \) in the trapezoidal approximation to \( \int_{a}^{b} f(x)dx \) satisfies \( |T E_{n}| \leq \frac{M(b-a)^3}{12n^2} \) where \( |f''(x)| \leq M \) for all \( x \) in \([a,b]\) and \( n \) is the number of subintervals. Use this bound to determine how large \( n \) must be in order to guarantee that \( T_{n} \) differs from \( \int_{1}^{2} 4\sqrt{x}dx \) by at most 0.0005.
Please read the following and sign below.

I have upheld the Duke Community Standard in completing this examination.

signature_________________________