Math 41.03 Fall 2003
Test #1

Name: ____________________________

Read all of the following information before starting the test:

- Be sure that this test has 8 pages including this cover.
- There are 9 problems on this test worth a total of 100 points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are not permitted on this exam.
- Show all work neatly and in order and indicate your final answers clearly.
- Answers must be justified whenever possible in order to earn full credit. No credit will be given for unsupported answers, even if your final answer is correct.
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- I have adhered to the Duke community standard in completing this test ________________.
- Good luck!

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1. (6 points) State the Fundamental Theorem of Calculus.

2. (10 points)
   (a) Find
   \[ \frac{d}{dx} \int_1^{x^4} \sec(\sqrt{t + t^2}) dt. \]

   (b) Find a formula for \( f(x) \) assuming that
   \[ x^2 = \int_x^1 \sqrt{t - 6f(t)} dt. \]
3. (12 points) Evaluate the following expressions by expressing the limits as definite integrals and evaluating the integrals.

(a) \[ \lim_{n \to \infty} \frac{\sqrt{1} + \ldots + \sqrt{n}}{n^{\frac{3}{2}}} \]

(b) \[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \sec^2 \left( \frac{i}{n} \right) \]

4. (12 points) Differentiate the following functions.

(a) \[ f(x) = \ln \left( (2x^2 - 5x)^{4x} \right) \]

(b) \[ f(x) = e^{\sec^2 x} \]

(c) \[ (3x^4 - 6x^2 + 5)^2(2x^3 - 9x + 1)^{\frac{1}{2}} \]
5. (14 points) If the graph below represents $f'(x)$ sketch a graph of what $f(x)$ might look like on the set of axes above $f'(x)$ and sketch $f''(x)$ on the set of axes below. Label all critical points and inflection points of $f(x)$. 

\[ f'(x) \]

\[ f''(x) \]
6. (10 points) Calculate the volume of the solid obtained by rotating the region between \( x = y^3 \) and \( y = x^2 \) about the line \( y = 6 \).

7. (8 points) Prove that the average rate of change of \( f(x) \) on \([a, b]\) \( (= \frac{f(b) - f(a)}{b-a}) \) is equal to the average value of \( f'(x) \) on \([a, b]\).
8. (14 points) Consider the integral $\int_{1}^{x} \frac{1}{t} dt = \ln x$.

(a) Using the fact that the trapezoid rule error $|ET_n|$ is bounded by

$$|ET_n| \leq \frac{M_2(b-a)^3}{12n^2},$$

determine the number of steps required to approximate $\ln x$ with error less than 0.001. (Note: your answer should depend on x.)

(b) Approximate $\ln 4$ using the trapezoid rule with $n = 3$ and determine a bound on the error in your approximation.
9. (14 points) Consider the region between the curves $y = 2x^2$ and $y = x^2 + 4$.
(a) Calculate the area of this region.

(b) Calculate the volume of this region rotated about the $y$-axis.
Scrap Page

(You may carefully remove this page from the test booklet.)