9. (5 points) Use the trapezoidal approximation to estimate the volume of a hill if the areas of horizontal cross-sections of the hill at 10-ft. height intervals are as follows:

<table>
<thead>
<tr>
<th>Height x (ft.)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (ft.²)</td>
<td>1020</td>
<td>550</td>
<td>75</td>
<td>0</td>
</tr>
</tbody>
</table>

You may leave your answer as an indicated sum.

\[
V = \int_{0}^{30} A(x) \, dx = T_3 = \frac{10}{2} \left( 1020 + 2 \cdot 550 + 2 \cdot 75 + 0 \right) \\
= 5 \left( 1020 + 1100 + 150 \right) \\
= 5 \left( 2270 \right) = \boxed{11350 \text{ ft.}^3}
\]

10. (11 points) (a) Recall that the error \( |ET_n| \) in the trapezoidal approximation to \( \int_{a}^{b} f(x) \, dx \) satisfies

\[|ET_n| \leq \frac{M_2(b-a)^3}{12n^2}, \text{ where } |f''(x)| \leq M_2 \text{ for all } x \text{ in } [a,b] \text{ and } n \text{ is the number of subintervals. Use this upper bound for } |ET_n| \text{ to determine how large } n \text{ must be in order to guarantee that } T_n \text{ differs from } \int_{1}^{3} \left( \frac{x^2 + \frac{1}{x}}{2} \right) \, dx \text{ by at most } 0.005.

First find \( M_2 \).

\[f(x) = \frac{x^2}{2} + \frac{1}{x} ; \quad f'(x) = x - \frac{1}{x^2} ; \quad f''(x) = 1 + \frac{2}{x^3} ; \quad f'''(x) = -\frac{6}{x^4}.\]

Now \( f'''(x) < 0 \text{ on } [1,3] \Rightarrow f'''(x) \text{ is decreasing on } [1,3].

Since \( f'''(x) > 0 \text{ on } [1,3] \), \( |f'''(x)| \leq f'''(1) = 3 \text{ on } [1,3].

Let \( M_2 = 3 \). Then \( |ET_n| \leq \frac{3 \cdot 2^3}{12n^2} = \frac{2}{n^2} \). Find \( n \) so that

\[\frac{2}{n^2} \leq 0.005 = \frac{1}{200} \quad \text{or} \quad \frac{n^2}{2} \geq 200 \quad \text{or} \quad n^2 \geq 400.
\]

\[n \geq 20\]

(b) Is \( T_n \) an underestimate or an overestimate for \( \int_{1}^{3} \left( \frac{x^2 + \frac{1}{x}}{2} \right) \, dx \)? Justify your answer.

\[f''(x) = 1 + \frac{2}{x^3} > 0 \text{ on } [1,3]. \text{ So } f(x) \text{ is concave up on } [1,3] \text{ and } T_n \text{ is an overestimate.} \]