MATH 563: Applied Computational Analysis
Spring 2020 syllabus

Instructor/Office: Jeffrey Wong (Physics 029B)

Office Hours: See course website for updated office hours

Times/location: M/W 3:05-4:20, Gross Hall 304B. Dates: **Jan. 8** to **Apr. 22**.

Course Website: Piazza site and Course Sakai site (for grades, emailed announcements)

Textbook: Main text: Quarteroni, Sacco and Saleri, *Numerical Mathematics*, 2nd edition. (may be available online). Secondary texts (purchase not necessary): Ascher and Petzold, *Computer Methods for Ordinary Differenial equations*; Leveque, *Finite Difference Methods for Ordinary and Partial Differential Equations*, 2007

Course description: An introduction to numerical methods for functional approximation, differentiation and integration, ordinary differential equations and related topics. The course will emphasize an understanding of numerical methods and their properties from the perspective of theory and practice and the interplay between the two. First, we briefly cover the essentials of approximation theory. Second, we study numerical analysis of dynamical systems. Last, a selection of more advanced topics will be considered, such as finite difference schemes for partial differential equations and/or stochastic differential equations.

Prerequisites: An solid grasp of undergraduate linear algebra and differential equations (e.g. Math 221 and Math 356) is essential. Experience with programming of some kind is also expected (be comfortable with writing code to implement algorithms). Numerical linear algebra (e.g. Math 561) is suggested, but not necessary (the relevant points will be reviewed as needed).

Exams and Grading: Your grade will be based primarily on homework, which includes both written work and code (see below). In addition, there will be a midterm about six weeks into the course. There is **no final exam**.

Homework:

- Homework will be assigned weekly, except before midterms and the final exam. Due dates will be listed on the assignment; typically one week after assigned.
- Homework should be turned in by the deadline to ensure full credit. If you miss a deadline, you should still complete the homework and turn it in for feedback (and partial, if not full, credit). The lowest homework score will be dropped.
- Working and studying in groups is strongly encouraged, as well as discussing code (writing code to be read by others is an excellent way to improve computational

skills!). However, the final product - solutions and code - should be your own.

- Guidelines for code and computational results are posted to the course website.
- Solutions should be complete arguments. Be thorough, but strive for clarity without extraneous work. Support work with computations when appropriate.
- Typing solutions (in LaTeX) is suggested; if handwritten, make sure solutions are readable. Keep problems in the same order as the assignment, if possible.

Overview of topics

The goal is to cover most if not all of the topics in the main areas listed. The details may change depending on time and interest.

- **Part I (approximation):**
  - Interpolation: polynomials, splines, Chebyshev
  - Orthogonal polynomials and Fourier series, FFT
  - Continuous least squares and min-max approximation
  - Derivatives (finite differences) and integrals (Newton-Cotes; Gaussian)
  - More (TBD; e.g. singular integrals,...)
- **Part II (ODE initial value problems):**
  - Theory: convergence and stability; stiffness and absolute stability
  - One step methods: Euler, Runge-Kutta methods (implicit and explicit)
  - Multi-step methods: Adams methods, BDF, predictor-corrector
  - Stochastic DEs, further topics in ODEs
- **Part III (ODE boundary value problems):**
  - Theory for boundary value problems; ODE vs. BVP stability
  - Shooting methods (basic, splitting up the interval, backward)
  - Finite differences (plus solving banded linear systems)
- **Part IV (PDEs):**
  - Finite differences for Poisson's equation;
  - Parabolic (heat-like) equations: method of lines; finite differences
  - Stability constraints (in practice; brief introduction)
  - Spectral methods (FFT for Poisson, ...)