On code: No code is to be turned in. You should, however, write code for divided differences (or use the one in the book; copying is fine). Work does not need to be shown in computing interpolating polynomials.

I suggest doing a few steps by hand until you are confident with the algorithm, but this is up to you (since the calculations are tedious).

Book problems

• Chapter 10: 3, 9, 16, 19 (Note: ‘spectral’ accuracy means the error may start out poorly, but will eventually decay exponentially with $n$, i.e. $\sim r^{-n}$).
• Chapter 14: 1, 2. For (b) and (c), provide both a theoretical answer and numerical evidence to verify that it is true.
• Note for 10.4.9: The text may have some punctuation missing. You should compute two quadratic interpolants, one using 0, 7, 14 and the other using 7, 14, 21.

Other problems

P1. A function $f(x)$ has the following data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1.9502</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.7769</td>
</tr>
<tr>
<td>0.3</td>
<td>3.0596</td>
</tr>
<tr>
<td>0.7</td>
<td>9.5662</td>
</tr>
<tr>
<td>1</td>
<td>22.0855</td>
</tr>
</tbody>
</table>

It is known that $f$ has a single zero $x^*$ in $[-1, 1]$.

a) Using three $x_i$'s as nodes and a quadratic interpolant, estimate $x^*$.

b) Find the interpolant for all of the data $(y_i, x_i)$, using the $y_i$’s as the nodes, and use it to estimate the value of $x^*$ (this is called inverse interpolation). Hint: Compute $p(0)$.

c) Assuming that all the derivatives of $f$ are known to be bounded in size by 10, give a reasonable estimate for a bound on the error in (a) and (b).

d) Under what conditions does inverse interpolation work?
P2. Let \( L_i(x) \) be the \( i \)-th Lagrange basis polynomial for \( n + 1 \) nodes \( x_0, x_1, \ldots, x_n \). For any function \( f(x) \), consider the approximation

\[
q(x) = \sum_{i=0}^{n} f(x_i)(L_i(x))^2.
\]

a) What is the degree of \( q(x) \)?

b) Show that \( q \) has the property that if the function \( f(x) \) is positive for all \( x \), then \( q(x) \) is also positive for all \( x \). \(^1\)

b) Show by way of a simple example that the usual interpolating polynomial does not have this property.

\(^1\)This is called ‘positivity’, which is sometimes desirable for an approximation scheme when the sign is important (such as when negative values of \( f(x) \) are not allowed).