Reading: Read and study the following (book sections refer to Boggess & Arnold).

- Review Sections 7.3-7.6 of the book (or equivalent linear algebra review). Useful problems (not required or to be turned in) are 7.4.18-21, 7.6.35, 7.6.36.

- [suggested] Section 9.1

Problems: Complete the following problems (‘Section’ refers to the textbook; for this week the book problems are also listed at the end.)

- Section 2.4: 13, 15, 18, 23 (For 23: use the method in problem 22, i.e. the substitution $z = 1/y$ but you don’t need to do problem 22 itself.)

- Section 9.2: 3, 9, 19, 44, 52 (Note: 9 and 52 use the solns from 3 and 44, respectively.)

- Problems P1-P2 (below)

**Non-book problems**

**P1.** For the homogeneous linear system $\mathbf{x}'(t) = A(t)\mathbf{x}(t)$ do the following:

a) Prove the superposition principle (i.e. linear combinations of solutions are solutions).

b) Show that it does not hold for an inhomogeneous system $\mathbf{x}' = A(t)\mathbf{x} + f(t)$.

**P2 (some linear algebra review).**

a) Suppose an $n \times n$ matrix $A$ has linearly independent eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_n$. Let $V$ be the matrix whose columns are the eigenvectors. Show that

$$A = VDV^{-1}$$

where $D$ is the diagonal matrix of eigenvalues. *Hint: multiply by $V$ on the right.*

b) What is the difference between the *algebraic* and *geometric* multiplicities of an eigenvalue?

c) Find the eigenvalues/vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$  

Is there a basis of eigenvectors of $A$ for $\mathbb{R}^3$?

d) Let $\mathbf{v}_1$ be an eigenvector of $A$. Find vectors $\mathbf{v}_2$ and $\mathbf{v}_3$ such that

$$\mathbf{v}_1 = (A - I)\mathbf{v}_2, \quad \mathbf{v}_2 = (A - I)\mathbf{v}_3.$$  

Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ form a basis for $\mathbb{R}^3$. *Note: These are ‘generalized eigenvectors’.*