MATH 356, FALL 2018
MIDTERM I GUIDE

Notes

This document provides information about the first midterm (on Tuesday, Oct. 2), the list of topics and suggested resources (problems) for studying. The midterm covers material through the Tuesday before the exam; a few topics may be omitted if we do not get to them or added if we do. Note that only a subset of the listed sections in Chapters 4 and 9 will appear on the exam (we covered Section 8.5, then the first part of Chapter 9, then moved to Chapter 4).

Formulas provided

The exam is closed book and closed notes. You will be given the following formulas (without context) if they are relevant to the exam problems:

- The variation of parameters formula.
- Any relevant trig identities or tedious integrals
- What to guess for undetermined coefficients (but you will have to know what to do from the hint), similar to the table on page 172 (Table 1: the method of undetermined coefficients).

Topics

Section numbers refer to the book, although it may not always be the best resource (these topics are also covered in the lecture notes). The notation will match that used in class and the lecture notes.

Topics marked in red will not be on the midterm. There will be no proofs but you may be asked to show something (similar to homework, but typically a little more straightforward).

Formulas are much less important to know than the ideas and process behind them. Other than what is listed, specific solution tricks are also not important. For instance, you should know how to change variables to transform an ODE, but you do not need to know to take $z = y^{1-n}$ to solve a Bernoulli equation.

- Solution techniques (know how to recognize certain equations and obtain solutions; general solutions and initial value problems)
  - Separable equations (2.2)
  - 1st order linear equations: Integrating factors (2.4)
  - Exact equations (2.6)
    - integrating factors for exact equations
Substitution: Given a change of variables, be comfortable with using it to transform the ODE into something easier to solve (no specific tricks).

Linear systems (9.1-9.5)
- Planar systems (all the cases)
- In higher dimensions (but not repeated roots)
- Matrix exponentials
- Further topics (from the Tuesday before the exam)

$n$-th order linear ODEs (4.1-4.3, 4.5-4.6, 9.8)
- Second order, const. coeff. (all the cases, so given any random one you should be able to solve it). Sections 4.1-4.3 and 4.5-4.6
- $n$-th order const. coeff. ODEs (all the cases; note that finding the eigenvalues is hard to do by hand unless it factors nicely)
- Reduction of order
- Other specific techniques for non-constant coeff. linear ODEs

Inhomogeneous ODEs (4.5-4.6 for second-order, 9.9 for systems)
- Variation of parameters (you’ll have the formula; know how to use it. Note that this is tedious to compute except in nice cases)
- Undetermined coefficients for $n$-th order ODEs (know the idea and what to guess; for anything complicated you will have a hint; don’t bother with this for general linear systems). Note that only the second-order case is reasonable to compute by hand.

Concepts and qualitative behavior: fundamental theory of ODEs and what we can say about solutions with or without solving the ODE explicitly. This includes long term behavior (what happens as $t \to \pm \infty$), constraints on where solutions can go and how far they extend and, most importantly, the structure to solutions of linear ODEs.

Linearity (most of this is from the lecture notes and is scattered through the book)
- Definition of a linear operator (as relevant to ODEs), vector space, linear vs. non-linear ODEs (know the difference)
- homogeneous vs. inhomogeneous ODEs (what is the difference?)
- particular solutions; vector space structure of the homogeneous solutions (8.5)
- Basis for homogeneous solution space, linear independence of solutions for linear systems and for $n$-th order linear ODEs (8.5)
- Connection between linear systems and $n$-th order ODEs (know how the results for linear systems correspond to the results for $n$-th order ODEs)
- Verifying that solutions form a basis (via the linear independence result and/or the Wronskian, which is the same thing) (4.1 for second order; 8.5 for systems)
○ Existence and uniqueness (2.7)
  - First-order ODEs: Existence/uniqueness theorem
    - (know how to apply it, what it tells you)
    - Partial derivative bound / continuity condition for uniqueness
    - Lipschitz condition
    - Extension part of the theorem
    - Dependence on initial conditions lemma
    - Constraints on solutions (non-intersection)
  - Interval of existence (finding it by solving the equation); recognizing when solutions blow up / where they fail to exist
  - Non-uniqueness (be able to recognize this; understand the examples)
  - Linear system existence/uniqueness (the theorem in 3.2)
○ Autonomous equations (2.9)
  - Phase lines (drawing, using them to describe behavior)
  - Stable/unstable/half-stable equilibria; derivative condition
  - Use the phase line to describe qualitative behavior (increasing/decreasing; solutions staying between equilibria; limit as $t \to \pm\infty$
  - Bifurcation diagrams (i.e. know how to describe qualitative changes as a parameter varies as in the homework problems; you don’t need to memorize the types of bifurcations)

• Miscellaneous
  ○ Euler’s method

Suggested problems

Below is a list of useful textbook problems (to be updated depending on Tuesday’s class). Ones that I think are particularly helpful are marked with a star. Many of these problems are just equations to solve for practice with computation (some are a bit tedious, but are useful to do: factoring cubic polynomials, finding eigenvalues of $3 \times 3$ matrices and anything with systems of dimension 3 or more in particular).

Note that if you want to check your answer, a computer algebra package like Wolfram Alpha should be able to compute solutions.

• Section 2.2: 1-12, *13-22
• Section 2.4: 1-11, *12, 14-21, *27, 28 (you can ignore the hint)
• Section 2.6 9-21, 23, 25, *28, *30, 42, 43
• Section 2.7: 11-14b, 23, 28-30
• Section 2.9 30, 15-22(i,ii,iii)
• Section 3.1: *20, 21 (this is longer than an exam problem would be, but is a good exercise)
• Section 4.1: 1-8
• Section 4.3: 1-24, 25-36
• Section 4.5: 1-8, 18-23 (some of these may be tedious)
• Section 4.6: 1-6, 13
• Section 8.5: *17b, 23-26 (ignore the direct substitution part)
• Section 9.1: 16-23
• Section 9.2: 41-56
• Sections 9.3-9.5: (to be added, depending on the schedule)
• Section 9.8: 7-12 (note that linear independence is tedious to check for the $3 \times 3$ matrix), *13, *16, *25 (any of the others in this section are also good to solve; on an exam, for degree 3 and above you’d be given at least one root).
• Section 9.9: 1-6 (again, the actual calculations can be tedious)