**HOMEWORK 1**  
**MATH 356, FALL 2018**  
**DUE TUESDAY, SEP. 4**

**Instructions:** Complete the listed problems (including the additional problems). Sections refer to the textbook.

**Problems:**
- Section 2.2: 2, 11, 16, 17 (For 17, also sketch the solution on its interval of existence).
- Section 2.4: 1, 6, 21 (For 21 also answer: can you tell what the interval of existence should be from the ODE itself?)
- Also in section 2.4: 22, 23. *(This is an example of a ‘change of variables’ trick to turn a nasty ODE into a nice one).* You should compute $z'$ according to the hint, then use the ODE to write the result in terms of $x$, then replace all instances of $x^{1-n}$ with $z$).

**Additional problems:**

**P1.** (Completing an example from class) Find the interval of existence for the solution to

$$y' = (1 - 2x)y^2, \quad y(0) = y_0$$

as a function of $y_0$. Is it ever true that the solution exists for all $x \in \mathbb{R}$? *Hint: there are three cases, depending on $y_0$.*
List of book problems

Note: Provided for this week only.

1. Section 2.2

2. Find the general solution to $xy' = 2y$.

11. Find the general solution to $3y' = x + 2y'$.

16. Find the exact solution to the IVP

$$y' = e^{x+y}, \quad y(0) = 0.$$  

and its interval of existence.

17. Find the exact solution to the IVP

$$y' = 1 + y^2, \quad y(0) = 1.$$  

and its interval of existence. Sketch the solution on its interval of existence.

Section 2.4

1. Find the general solution to $y' + y = 2$.

6. Find the general solution to $tx' = 4x = t^4$.

21. Find the solution to the IVP

$$(1 + t)x' + x = \cos t, \quad x(-\pi/2) = 0$$  

and determine its interval of existence. Sketch the solution. Can you deduce the interval of existence from the ODE itself?

22. Text slightly rewritten; problem unchanged. The first-order ODE

$$x' = a(t)x + f(t)x^n, \quad n \neq 0, 1$$  

is known as Bernoulli’s equation. It is non-linear, so we cannot use the integrating factor technique directly. However, by a change of variables, we can transform this into a linear equation.

Show that the change of variable $z = x^{1-n}$ transforms the Bernoulli equation into

$$z' = (1 - n)a(t)z + (1 - n)f(t)$$  

which is linear. Hint: First show that

$$\frac{dz}{dt} = \frac{dz}{dx} \frac{dx}{dt} = (1 - n)x^{-n} \frac{dx}{dt}.$$  

Then use the ODE to write the result in terms of $x$ and replace all factors of $x^{1-n}$ with $z$.

23. Use the technique of the previous exercise to transform the Bernoulli equation

$$y' + y/x = xy^2$$  

into a linear equation. Find the general solution to the resulting linear equation (and then the solution to the original Bernoulli equation).