# HOMEWORK 8
# MATH 353, FALL 2020

DUE WEDNESDAY OCT. 21

*Supplementary problems:* Book, Section 10.1, 1-10 and 14-19.

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**Non-book problems:**

**P1.** By guessing solutions of the form $u(x,t) = f(t)g(x)$, find as many solutions as you can (preferably all of them!) to the heat equation

$$u_t = ku_{xx}.$$

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**P2 (some eigenvalue problems).** Solve the following eigenvalue problems:

$$\begin{aligned} \text{a)} \quad & -\phi'' = \lambda\phi, \quad \phi(0) = 0, \quad \phi'(L) = 0 \\ \text{b)} \quad & \phi'' + 3\phi = 4\lambda\phi, \quad \phi(0) = 0, \quad \phi(2) = 0 \\ \text{c)} \quad & -\phi'' - 2\phi' = \lambda\phi, \quad \phi(0) = 0, \quad \phi(1) = 0. \end{aligned}$$

In each case, find the eigenvalues and eigenfunctions. Be sure to check all the cases (to verify that the eigenvalue/functions you find are the only ones).

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**P3 (periodic BCs).** Consider the eigenvalue problem with **periodic** boundary conditions,

$$-\phi'' = \lambda\phi, \quad \phi(0) = \phi(2\pi), \quad \phi'(0) = \phi'(2\pi). \tag{1}$$

Find the eigenvalues and eigenfunctions.

In doing so, show that for each $\lambda$, the expression you get for $\phi$ can be written as the span of two familiar functions that are orthogonal in the inner product

$$\langle f, g \rangle = \int_0^{2\pi} f(x)g(x)\,dx.$$

*Hint: The 'determinant' check will be useful here, as the system does not appear to simplify well. The determinant simplifies with the use of some trig. identities. Getting orthogonality should not require much extra work after solving the equation.*

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**P4 (orthogonality).** Check that the functions $\phi_n = \sin(nx)$ (for $n = 1, 2, 3, \cdots$) are orthogonal in the inner product

$$\langle f, g \rangle = \int_0^{\pi} f(x)g(x)\,dx.$$

Also calculate $\langle \phi_n, \phi_n \rangle$. *Hint: use the 'product' identity for sines, and the* $\sin^2$ *identity - consult the usual trig. identity tables.*