# HOMEWORK 7
# MATH 353, FALL 2020

DUE WEDNESDAY OCT. 14

**Reading and Book Problems:**

- Section 6.4: 8b
- Section 6.5: 12
- Section 6.6: 21

*Supplementary problems:* As before, problems 1-8 of 6.4 and 1-8 of 6.5 are good 'random' problems to pratice with the procedure. For convolutions, consider 7-9 and 11-15 and for series solutions, 1-8 and 18.

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**Non-book problems:**

**P1.** Verify, by use of the ratio test, that the power series solutions for the Airy equation (around $x = 0$) converge everywhere.

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**P2 (series solution).** Here we consider power series solutions to the IVP

$$y'' - y = x^3, \quad y(0) = 1, \quad y'(0) = 1.$$

Look for a power series solution around $x_0 = 0$. *Note: use the initial conditions to determine $a_0$ and $a_1$; be careful with the $x^3$ term.* Obtain the recurrence relation and find the solution up to and including the $x^5$ term.

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**P3 (a singular example).** Suppose you want to find a power series solution to

$$x^2y'' + (1 + 3x)y' + y = 0.$$

a) Look for a solution $y(x) = \sum_n a_n x^n$ and derive the recurrence formula for the coefficients. Solve it. How many linearly independent solutions do you get out of this calculation?

b) What is the radius of convergence for this solution?