HOMEWORK 4
MATH 353, FALL 2020

DUE WEDNESDAY, SEP. 16

Book Problems:

- Make sure that you can do the problems 1-15 in 3.3 and 1-11 in 3.4 - none are assigned, but you should be comfortable with the mechanics of solving LCC ODEs.
- Section 3.4: 10
- Section 3.5: 1, 5 *Note: find the general solution, not just a particular one.*

Supplementary problems: 3.1: 21. 3.2: reading the section. 3.3: Any of 1-15.
3.4: any of 1-11, 16, 17. 3.5: reading the section, 1, 5, 8, 9, 23.

Non-book problems:

P1. In discussing the procedure for solving **LCC second order ODEs**, it was assumed that the solutions found were linearly independent ($e^{\lambda t}$'s etc.). For each of the three cases, show that the pair of basis solutions is actually linearly independent.

P2 (more on LI solutions). Note: for both parts below, call an ODE 'nice' if its coefficients are all continuous functions (e.g. $y'' + t^3 y = 0$ but not $y'' + y/t^3 = 0$).

a) Consider the two functions $f(t) = t$ and $g(t) = \sin t$. Can these two functions be solutions to a nice second order, linear, **homogeneous** ODE (**not necessarily LCC**)? *Hint: P1 may be helpful; don't try to construct an ODE explciitly like in P3.*

b) Can $y(t) = t^3$ be the solution to a nice second order, linear, **homogeneous** ODE?

P3. Assuming $m = 1$ (the mass), the spring ODE has the form

$$y'' + cy' + ky = 0.$$

a) Show that there is a 'critical' value c^* such that solutions oscillate when $0 < c < c^*$ ('underdamped') and do not oscillate when $c > c^*$ (overdamped). What happens for $c = c^*$?

b) Show that all solutions tend to zero as $t \to \infty$ when the damping is positive ($c > 0$). What happens if $c < 0$, and does the solution make sense in the context of the model?

P4. a) Create an LCC ODE that has a solution $y(t) = 7e^t \sin 3t$.

b) Create a third order, LCC ODE whose general solution is

$$y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{3t}.$$