HOMEWORK 3
MATH 353, FALL 2020

DUE WEDNESDAY, SEP. 9

(Due before midnight on Wednedsay. I suggest doing P4 before Monday's class).
Correction: P2 (noted in red).

Book Problems:

- Section 2.6: 4, 5, 11

Supplementary problems: 2.5: 6-9, 19, 2.6: 9, 10, problems 1-24 in the chapter review after chapter 2 (this tests if you can recognize types of DEs).

Non-book problems:

P1 (a bifurcation). Consider the ODE

$$y' = ry + y^3$$

where r is a real number. The qualitative behavior of the ODE depends on r.

a) Identify the ranges of r for each case (as done in lecture for the fish example with H) and draw the phase line(s). *Note: you don't have to use a graphical argument as in class, and it's probably easier to do some calculation instead.*

b) For each case from (a), find the initial values y_0 for which $\lim_{t\to\infty} y(t) = 0$.

P2 (doing P1 the hard way). Find an explicit formula for $y(t)$ solving

$$y' = y - 4y^3, \quad y(0) = 1$$

Hint: use partial fractions. Simplify as much as possible (to solve for y explicitly).
Does the limit as $t \to \infty$ match the phase line analysis (as done in class)?

P3. Note: to be precise, an equation is 'separable' if it can be rearranged into separable form; the same goes for an exact equation.

a) Is every separable ODE an exact ODE? If yes, explain; if no, provide a counter-example.

b) Is every exact ODE a separable ODE? If yes, explain; if no, provide a counter-example.

(continued on page 2)

P4 (real and imaginary parts of solutions). Consider the ODE

$$y'' + p(t)y' + q(t)y = 0$$

where $p(t), q(t)$ are **real-valued** functions.

a) Suppose $z(t)$ is a complex-valued solution (e.g. something like e^{it}). Show that its real and imaginary parts are also solutions.

Hint: write $z(t) = v(t) + iw(t)$*; also note that* $a + bi = c + di$ *if and only if* $a = c$ *and* $b = d$.

b) Does your argument work for either of the following ODEs?

$$\text{(i)} \quad y'' + iy = 0.$$

$$\text{(ii)} \quad y'' + y^2 = 0.$$

If not, identify the part of the argument that fails.