# HOMEWORK 10
# MATH 353, FALL 2020

DUE WEDNESDAY NOV. 11

**Note:** You may use computer algebra (e.g. Wolfram alpha) to do the Fourier integrals. Make sure the steps are complete. otherwise. Book problems:

- Section 10.3: 2
- Section 10.4: 17b, 18b (do P1 first; you do **not** need to compute the Fourier series)
- Section 10.7: 1a; you may assume $L = 1$

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**Non-book problems:**

**P1 (extensions).** Suppose that $f(x)$ is a function defined on $[0, \ell]$ - think of this as half of the "Fourier series interval" $[-\ell, \ell]$. We can define an **even extension** to this interval by

$$f_e(x) = \begin{cases} f(x) & 0 < x < \ell \\ f(-x) & -\ell < x < 0 \end{cases}$$

to get a function on $[-\ell, \ell]$ as used in class.

Similarly, we can construct an **odd extension**

$$f_o(x) = \begin{cases} f(x) & 0 < x < \ell \\ f(-x) & -\ell < x < 0 \end{cases}$$

This gives us a function on $[-\ell, \ell]$ that can then be interpreted as $2\ell$-periodic.

a) Suppose

$$f(x) = x^2, \quad x \in [0, 1].$$

Sketch the odd and even extensions of $f(x)$ to $[-1, 1]$ and then extend those functions to be period 2 (draw over a few periods).

b) What can you say about the Fourier series for the even/odd extensions of a function?

c) Observe that the series for even/odd extensions for (b) can **both** be used to represent the same function $f(x)$ on $[0, \ell]$. Where have you seen these representations before?

**P2 (wave equation, with nonhomogeneous BCs).** A string is fixed at both ends, then made to move by oscillating one end up and down with a frequency 2. Suppose the IBVP describing this is

$$u_{tt} = 9u_{xx}, \qquad x \in (0, \pi),\ t > 0$$
$$u(0,t) = 0, \quad u(\pi, t) = \pi A \sin 2t$$
$$u(x,0) = 0, \quad u_t(x,0) = 0$$

a) Let

$$w(x,t) = Ax \sin 2t$$

and let $v = u - w$. Show that

$$v_{tt} = 9v_{xx} + f(x,t)$$

for a certain function $f$ (hint: plug $u - w$ into the equation above - then solve for $f$).

b) Now show that $v$ satisfies **homogeneous** BCs, and determine the initial conditions for $v$ (using superposition).

c) Finally, solve the problem for $v(x,t) = \sum c_n(t)\phi_n(x)$. Write down the solution for the $n = 2$ term explicitly (you can leave the rest of the coefficients as solutions to IVPs that could be solved).

What happens after a moderate amount of time (i.e. as $t$ becomes large)?

Note that you'll need to compute the expansion

$$x = \sum_n a_n \phi_n(x)$$

and you can leave formulas in terms of explicit integrals except for the $n = 2$ term as requested.