HOMEWORK 1
MATH 353, FALL 2020

DUE FRIDAY, AUG. 28

Instructions: Complete the listed problems - a mix of problems from the textbook (the ones labeled as 'Section....') and non-book problems (labeled as Px). Homework is due on the stated day before class (3:30 PM).

Note: For separable equations, leave solutions in implicit form if they are not reasonable to solve explicitly, e.g. if a solution $y(x)$ satisfies $\sin(y) + 3xy = C$ or $y^8 + 2y^4 + 1 = x$.

Book Problems:

- Section 2.1: 5c, 6c, 12
- Section 2.2; 7, 10(ac), 18

(useful practice: any nearby problems to the ones assigned)

Non-book problems:

P1. The ODE

$$y' = x^2 y$$

can be solved in several ways.

a) Solve for $y(x)$ using an integrating factor.

b) Solve for $y(x)$ by separating variables. Show that you get the same solution as in (a).

c) What, in general, does a first-order ODE look like if it can be solved both by an integrating factor and by separating variables?

P2 (variant of 2.1.23). Suppose a, b and λ are constants, $b \neq 0$ and

$$y' + ay = be^{\lambda t}.$$

a) Show that if $a > 0$ and $\lambda < 0$ then **all solutions** to thie ODE go to zero as $t \to \infty$.

b) For what values of a, λ is thhe limit as $t \to \infty$ a non-zero (finite) value?

c) Is it true that if $a < 0$ and $\lambda < 0$, then all solutions go to $\pm\infty$ as $t \to \infty$? Explain.

P3. We defined an 'initial value problem' for a first order ODE to be

$$y' = f(t, y), \quad y(a) = b, \tag{1}$$

specifying the value of y at a time $t = a$. One could also specify other 'initial' values...

a) Consider the 'initial value problem'

$$y' = t + y^2, \quad y'(1) = 5.$$

In what way is this a different problem than (1)?
Can you convert this into a problem of the form (1)?

b) Consider the 'initial value problem'

$$y' = f(t, y), \quad y(a) = b, \quad y'(a) = c.$$

Does this IVP always have a solution? If no, give a simple condition on a, b and c under which it does not have one.

Book problems (transcribed):

2.1.5c. Find the general solution of

$$y' - 2y = 3e^t$$

and use it to determine how solutions behave as $t \to \infty$.

2.1.6c. Find the general solution of

$$ty' - y = t^2 e^{-t}, \quad t > 0$$

and use it to determine how solutions behave as $t \to \infty$.

2.1.12. Find the solution to the initial value problem

$$ty' + (t+1)y = t, \quad y(\ln 2) = 1, \quad t > 0$$

2.2.7. Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{x}.$$

2.2.10ac. a) Find, in explicit form, the solution to the initial value problem

$$y' = (1 - 2x)/y, \quad y(1) = -2.$$

c) Determine the interval in which the solution is defined.

2.2.18. Solve the initial value problem

$$y' = \frac{3x^2}{3y^2 - 4}, \quad y(1) = 0$$

and determine the interval in which the solution is valid. (Hint: look for points where the solution has a vertical tangent).