This document provides information about the first midterm (on **Tuesday, Oct. 2**), the list of topics and suggested resources (problems) for studying. No new topics will be added beyond this list. The midterm covers material through the Tuesday before the exam; a few topics may be omitted if we do not get to them.

**Formulas provided**

The exam is **closed book and closed notes**. You will be given the following formulas (without context) if they are relevant to the exam problems:

- **Variation of parameters**:

  \[ y_p(t) = -y_1(t) \int_{t_0}^{t} \frac{y_2(s)g(s)}{W(s)} \, ds + y_2(t) \int_{t_0}^{t} \frac{y_1(s)g(s)}{W(s)} \, ds. \]

- Table 3.5.1 in the book (form of \( y_p \) for undetermined coefficients).

- Any relevant trig identities or tedious integrals

**Topics**

Section numbers refer to the book, although it may not always be the best resource (these topics are also covered in the lecture notes and **ODE and PDE Notes I** on Sakai). The notation will match that used in class and the lecture notes.

Topics **marked in red** will not be on the midterm. There will be no proofs but you may be asked to show something (similar to homework).

Formulas are much less important to know than the ideas and process behind them. Other than what is listed, specific solution tricks are also not important. For instance, you should know how to change variables to transform an ODE, but you do not need to know that \( y' = f(y/x) \) is transformed by taking \( v = y/x \).

- **Solution techniques** (know how to recognize certain equations and obtain solutions)
  
  - Separable equations (2.1)
  
  - 1st order linear equations: Integrating factors (2.2)
  
  - Exact equations (2.6)
    
    - integrating factors for exact equations
  
  - Substitution: Given a change of variables, be comfortable with using it to transform the ODE into something easier to solve (no specific tricks).
• Second order, const. coeff. linear ODEs (3.1-3.4)
  – Connection to linear systems (the context is helpful, but not required)
  – General solution, basis solutions in the various cases solving IVPs
  – Inhomogeneous case \( y_p \) using undetermined coefficients (know what to guess and how to get the solution; you’ll have the table as a reminder)

• Second order linear ODEs (3.4,3.6)
  – Variation of parameters to get \( y_p \)
  – Reduction of order to get \( y_2 \) from \( y_1 \) (set \( y_2 = vy_1 \))
  – other ways to solve specific ODEs with non-constant coefficients

• Concepts and qualitative behavior: fundamental theory of ODEs and what we can say about solutions with or without solving the ODE explicitly. This includes long term behavior (what happens as \( t \to \pm \infty \)), constraints on where solutions can go and how far they extend and, most importantly, the structure to solutions of linear ODEs.

• Linearity
  – Definition of a linear operator (as relevant to ODEs), vector space
  – Be able to recognize when an ODE is linear
  – homogeneous vs. inhomogeneous ODEs (what is the difference?)
  – particular solutions; vector space structure of the homogeneous solutions
  – Basis for homogeneous solution space, linear independence of solutions (3.2)
  – How to verify that solutions form a basis (via the Wronskian / linear independence)

• Existence and uniqueness
  – Difference between an ODE and an IVP
  – First-order ODEs: Existence/uniqueness theorem (2.4)
    · (know how to apply it, what it tells you)
  – Interval of existence (finding it by solving the equation)
  – Non-uniqueness (be able to recognize this; understand the examples)
  – Second-order linear existence/uniqueness (the theorem in 3.2)

• Autonomous equations (2.5)
  – Phase lines (drawing, using them to describe behavior)
  – Stable/unstable/half-stable equilibria
− Use the phase line to describe qualitative behavior (increasing/decreasing; solutions staying between equilibria; limit as $t \to \pm \infty$
− Describe how phase lines change as a parameter is varied (as in the fish example)

- Miscellaneous
  - Euler’s method (2.7)
  - Describing solutions (to any ODE) as $t \to \infty$ (and identifying when a solution blows up at an asymptote)

Suggested problems

Some general resources:

- Additional Homework Problems (all good for testing your understanding of concepts and definitions). Relevant problems: 1-38 (except 24, 27, 29, 31) and some of 39-45 (these may go past the material on the exam; see list above).

- Past midterms: available on Sakai (the shared site, not the Section 3 one). Midterms from the last two/three years will be the most relevant. Note that these midterms may include some material not on the exam (usually power series and Laplace transform).

Below is a list of useful textbook problems. Ones that I think are particularly helpful are marked with a star. Many of these problems (usually the first block in the section) are just equations to solve for practice with computation.

- Section 2.1: 1-20, *(21-23) (parts b and c), *31, 34, 35
- Section 2.2: *10, 12, 13, 14, 16
- Section 2.4: 24-26, *27, 28
- Section 2.5: 7, 9-13, *14, *25, 26, 27(a,b)
- Section 2.6: 1-14, *15-16, 21, 23
- Section 2.9 (the miscellaneous problems): 1-32, *33, 34a
- Section 3.1: 1-18, *27,*28
- Section 3.3: 1-22
- Section 3.4: 11-15, *37, *38
- Section 3.5: 1-12, 15-20 Note: undetermined coefficients problems often require substantial work but the process is straightforward; problems on the exam will not be too long. Also *33, *34 (this may use results from 37-39 in 3.4).
- Section 3.6: 1-12 (note that for variation of parameters, you really only need to know how to apply the formula once you have the homogeneous solution. Also note that except for simple problems, this can be a tedious calculation.) *21, *23.