The second midterm will take place on Thursday Nov. 15 in class. No new topics will be added beyond this list. The midterm covers material through the Thursday before the exam. No topics will be added; the list will be updated depending on Thursday’s class.

Formulas provided

The exam is closed book and closed notes. You will be given the following formulas (without context) if they are relevant to the exam problems:

- Trig identities for double/half angles and tedious integral formulas
- The Laplace transform table from the textbook
- A table with general partial fractions rules as needed
- The Fourier series formula (form of the series and coefficients). Note that you may need to do some variation where the formula does not apply directly.

Some advice

- **Review background** (see next section). We need to solve (simple) ODEs to solve PDEs!
- **Understand what you need to solve the problem.** We have some powerful general methods, but they take work. A complete solution is often not required to answer a question. For instance, if asked to show that all solutions
  \[ u(x, t) = \sum_{n=1}^{\infty} b_n e^{-\lambda_n t} \phi_n(x) \]
  to a certain heat equation problem converge to zero, it is enough to show that \( \lambda_n > 0 \) for all \( n \); finding \( b_n \) in terms of the initial condition is unnecessary.
- **Use computational shortcuts.** Use the tricks for integrating odd/even functions. If you know only a few modes will be present in a solution, write \( u(x, t) \) only in terms of those eigenfunctions instead of solving for all the (mostly zero) terms.
- **Be very careful with domains.** For convergence, the periodicity of the function can matter (if its endpoint values do not match). The formulas for the Fourier series, inner products and so on all depend on the interval.
- **If stuck on a computation,** just set it to a variable. For instance if you end up with
  \[
  \int_0^\pi x^2 \sin(nx) \, dx
  \]
set this equal to something (say, \( c_n \)) and move on. The same goes for Laplace transform (set transforms/inverses equal to something).

**Key Background**

**It is important** that you are comfortable with the practical calculations we learned for ODEs from the first half of the course and some miscellaneous tricks. This includes:

- **Partial fractions.** You will not need to use them for anything more than a cubic denominator, e.g.
  \[
  \frac{1}{s(s^2 + 2s + 2)}
  \]
  but be comfortable doing the computations for something like the above.

- **Integrating factors** first order linear ODEs. Such equations arise when solving for the coefficients \( a_n(t) \) in an eigenfunction series, e.g. equations like
  \[
  a_n' + \lambda_n a_n = t
  \]

- **Second order, constant coefficient ODEs** with a parameter. Given an equation like
  \[
  y'' + y' + ky = 0
  \]
you should be able to solve it (find the general solution) and identify the cases (complex roots, repeated, real and distinct) depending on \( k \) (see: eigenvalue problems).

- **Theory for linear ODEs** (linear independence, homogeneous/particular solutions, basis etc.). These concepts show up in series solutions and in the Laplace transform, since we are solving linear ODEs and need to use concepts like linear independence, bases etc.

**Topics**

All topics are covered in lecture notes. Section numbers refer to the book and NOTES I and NOTES II refer to ODE and PDE notes I and II. The notation will match that used in class and the lecture notes.

Topics marked in red will not be on the midterm.

- **Series solutions** (Book: 5.1-5.3; NOTES I 7.1-7.2)
  - Notable omissions:
    - Frobenius series; the indicial equation
    - Euler equations
  - Calculations:
    - Compute power series solutions for second/first order ODEs (some number of terms)
    - Identify ordinary and regular singular points
    - Calculate radius of convergence (directly)
- general, explicit solutions to the recurrence

○ Concepts:
  - Splitting the solution into the span of two basis solutions
  - Verifying linear independence
  - lower bound on radius of convergence (distance to nearest singular point)

- Laplace transform (Book: Section 6.1-6.6)
  ○ Notable omissions:
    - Theory: domain of the Laplace transform (exponentially bounded functions)
    - Theory for the dirac delta (definition using limits)
    - Deriving formulas from the definition (you should know this for the final but it will not be on the midterm)
  ○ Calculations:
    - Transform and inverse transform of various functions (as in the homework)
    - Solve $n$-th order constant coefficient ODEs using the Laplace transform
    - Use the convolution theorem to transform/inverse transform and solve IVPs
    - Solve ODEs with step functions and deltas

- PDEs and Eigenfunction series (Book: 10.1, 10.5, parts of 11.1); NOTES II 2.1-2.2 and 4.1-4.2)
  ○ Notable omissions:
    - Non-homogeneous PDEs (all PDEs on the midterm will be homogeneous)
    - Convergence theory for Fourier series (pointwise, $L^2$ convergence)
  ○ Calculations (PDEs)
    - Solve eigenvalue problems (including showing no solutions in various cases)
    - Solve the (homogeneous) heat equation in a bounded domain with nice boundary conditions using eigenfunctions
    - Determine coefficients using orthogonality of eigenfunctions
    - Detailed analysis of the eigenvalues in nasty cases ($\mu = \tan \mu$ etc.)
  ○ Concepts (PDEs)
    - Superposition and linearity
    - What it means to be an orthogonal basis for $L^2$, the inner product
    - Behavior as $t \to \infty$ (using the eigenvalues)
Sets of eigenfunctions form an orthogonal basis for functions that satisfy the boundary conditions (note: you do not have to know the details of the theorem).

- Fourier series

- Note that while Fourier series will not be on the exam, they are closely related to the eigenfunction series we use for the PDEs that are on the exam; the Fourier series is really an example of the eigenfunction material.

**Suggested problems**

- Additional Homework Problems:
  - **Series solutions**: 50-55 (only part (i) of 53)
  - **Laplace transform**: 56-60 (most were on HWs)
  - **Eigenfunctions, fourier series**: 63-69(i)-(iii), 70-72
  - **PDEs**: 73-77, 79,80

- Past midterms: available on Sakai (the shared site, not the Section 3 one). Midterms from the last two/three years will be the most relevant. Note that these midterms may include some material not on the exam (usually various PDE problems or extensions we have not yet covered; sometimes Frobenius series).

- Review problems (to be provided separately) and textbook problems (see below). Note that most of them are just normal problems to solve using the methods you are expected to know.

**Laplace transform:**

- Section 6.2: 1-23 (except 18,19)
- Section 6.3: 13-24, 32-33
- Section 6.4: 1-11; Section 6.5: 1-11
- Section 6.6: 4-20, 21.

**Series solutions:**

- Section 5.2: 1-18
- Section 5.3: 10, 18, 19, 22 (for 22: don’t bother finding the general formula)

**Fourier series:**

- Section 10.2: 13-18, 28
- Section 10.3: 1-4, 6 (the calculations can be tedious here)

**PDEs:**

- Section 10.1: 14-19; Section 11.1: 7-10(a-c), 19
- Section 10.5: 7, 8, 11
• Section 10.6: 15(ab) (just solve the problem using eigenfunctions), 20(a-c)