Instructions: Complete the problems below. Here ‘Section ...’ refers to the textbook and ‘AHP’ refers to the Additional Homework Problems document (posted on Sakai).

Text/AHP Problems:
- Section 2.4: 7, 9, 22
- Section 2.6: 5, 16, 25 (hint: find an integrating factor \( \mu(x) \)).
- AHP 6, 11, 27.

Other problems:
P1. a) Find the general solution to
\[ y' = y(1 - y). \]
b) Find the solution to the IVP
\[ y' = y(1 - y), \quad y(0) = y_0 \]
when \( 0 < y_0 < 1 \). Determine the limits \( \lim_{t \to \infty} y(t) \) and \( \lim_{t \to -\infty} y(t) \). Note that this is much more work than deducing the same limits from the phase line.

Second part.

Section 2.5: 3. Note: For the sketch, just make sure it is qualitatively correct (don’t solve the ODE exactly) - the right limits and increasing/decreasing behavior. Try to plot enough curves to show all the possible solution behaviors (e.g. like Figure 2.5.8).

AHP: 20.
P2. a) Explain (carefully) why solutions to
\[ y' = 1 - y^2, \quad y(0) = y_0 \]
must be increasing for all \( t \) if \(-1 < y_0 < 1\).
b) Corrected: Consider the IVP
\[ y' = (1 - y^2)^{1/2}, \quad y(0) = 0. \]
Show that \( y(t) = \pm 1 \) are solutions to the ODE and that \( y(t) = \sin(t) \) is a solution to the ODE for \( t \in [0, \pi/2] \) (take the positive square root). Thus, a solution is allowed to intersect \( y = 1 \). Why doesn’t the argument in (a) apply here?