MATH 353, FALL 2018
FINAL EXAM STUDY LIST (UPDATED DEC. 7)

The final exam will take place on Friday, Dec. 14 from 9AM to noon. The location is not in the normal classroom; the room is listed on DukeHub. The exam is closed book and closed notes. However, you will receive a formula sheet (posted to Sakai).

Some advice

• Make sure you are comfortable with the fundamentals (the starred items in the topics list) - both in computing solutions using these techniques and understanding the theory.

• Pay attention to what is required to solve a problem. Often, answering a problem does not require obtaining a 'full' solution. For instance, calculating a steady state may not require solving the PDE while proving it is a steady state does require obtaining the solution (although the coefficients do not have to be computed explicitly).

• Break longer problems into parts if necessary; setting unknown integrals, etc. to variables (e.g. Laplace transforms, Fourier coefficients). Compute them if you have time. This is important if you end up making a mistake that leads to unintentional computations.

• Know the essential terms (homogeneous, inhomogeneous, exact, linear etc.), what they mean, and how they are relevant to the various problems you need to solve. Note that some terms depend on context (e.g. linear operator vs. linear ODE).

• There are a few useful computational tricks to note. Odd and even symmetry, for instance, helps to compute Fourier integrals. Using a sinh / cosh basis vs $e^{\pm x}$ can make some (but not all) PDE problems easier. For key calculations, be aware of the efficient way to compute.

Topics

All topics are covered in lecture notes. However, it would be useful to look through the ODE and PDE notes document as well for another perspective. Topics marked in red will not be on the final. Topics with a star in blue ( *blue) are of central importance. Topics with a plus (+in purple) were skipped in the midterms but may be on the final.

• General concepts (that appear across several topics)
  ◦ Linearity (for operators, ODEs and PDEs)
  ◦ Superposition and linear combinations of solutions
  ◦ Linear independence and bases for a vector space (e.g. space of solutions to a homogeneous ODE; or for a space of functions as in Fourier series)
  ◦ General solutions vs. unique solutions (to IVPs / BVPs); how initial and boundary conditions determine arbitrary coefficients
Deducing qualitative behavior: Long-term behavior (what happens as \( t \to \infty \)), oscillations; leading-order behavior (e.g. exponential decay rate for the heat equation)

- **Fundamentals (ODEs)**
  - *Integrating factors and separable equations*
  - Existence and uniqueness for IVPs
  - Autonomous equations (phase lines; equilibria and stability)
  - *Second-order, constant coefficient ODEs (general solution; IVPs)*
  - Undetermined coefficients and variation of parameters
  - Linear independence and the Wronskian

- **Specific solution techniques (ODEs)**
  - Laplace transform
    - *Computing transforms and Solving IVPs*
    - Step functions, deltas, convolutions
    - +Derivation of some rules (e.g. derivative rule)
  - Series solutions
    - *Computing power series solutions; radius of convergence*
    - Definition of an ordinary point, regular singular points
    - Euler equations
    - the indicial equation, Frobenius series

- **Other ODE topics**
  - Reduction of order \((y_2 = vy_1\) to get a second solution)
  - Substitution tricks (the idea/process, not the specific substitutions)
  - +Exact equations
  - Numerical methods (Euler’s method)

- **Supporting theory (PDEs)**
  - \(L^2\) and *orthogonal bases*
  - *Fourier series (odd/even symmetry, computation)*
  - Fourier series (pointwise convergence theorem)
  - *Eigenvalue problems and boundary value problems*
  - Self-adjoint operators

- **Fundamentals (PDEs)**
Separation of variables

*Eigenfunction expansion method
  - Used to solve PDEs with source terms

*Finding/identifying the right eigenfunction basis

*Applying the methods to the heat, wave and Laplace’s equation

Specific solution techniques (PDEs)

Superposition/linearity tricks:
  - *steady states
  - *Solving PDEs with a finite number of modes
    - subtracting out boundary conditions (understand this but you don’t have to memorize the trick of guessing a linear function)
    - Splitting the problem into simpler ones (e.g. for Laplace’s equation)

Inhomogeneous BCs (directly, as in Section 4 of the notes)

Laplace’s equation in a disk (problems with periodic BCs)

Suggested problems

See previous midterm guides for book problems on earlier material. All the AHP’s are good for studying. For PDEs and Fourier series, here is a list of useful book problems:

Fourier series

- Section 10.2: 13-17, 19-22 (all just computation), *29
- Section 10.4: 15-18, 20-22, *(31-34) (even/odd function properties), *37

PDEs (fundamentals)

- Section 10.5: 7,8, *22, *23 (separation of variables process; 1-6 are also useful here)
- Section 10.6: 5-8, 12a, 13a, 20, 21
- Section 10.7: *9 (just solve the problem), *23a (parts (b)-(c) are interesting but not directly relevant to the exam)
- Section 10.8: 2,3, *4, *8(a,b), *10 (solve the problem, ignore (b))

PDEs (more topics)

- Section 10.8: 5-7 (Laplace in a disk)
- Section 11.1: 1-6, and 8,10 parts a-c (not part d!)
- Section 11.2: 1-13 (ignore the ‘normalized’ part)
- Section 11.3: 19, *22, 24