TEXT PROBLEMS:

- Section 5.4: 37
- Section 6.1: 6, 7, 27.  
  (Hint for 27: find a lower bound for the integrand and use Thm 6.1.1).
- Section 6.2: 9, 16, 21, 29.
- Section 6.3: 6, 17, 22, 25, 33 (For 33: write the solution in closed form).

OTHER PROBLEMS.

P1 (Failure of the Frobenius method). Consider the ODE\(^1\)

\[ x^2 y'' + (1 + 3x)y' + y = 0 \]

a) Show that \(x = 0\) is an irregular singular point.

Ignoring (a), try to apply the Frobenius method anyway by substituting

\[ y = \sum_{n=0}^{\infty} a_n x^{r+n} \]

into the ODE.

b) Show that \(ra_0 = 0\).

c) Find a recurrence for the coefficients and solve it. Show that if \(r = 0\) then we do obtain a non-trivial series (i.e. the coefficients are not all zero).

d) Show, however, that the radius of convergence of the series is zero.

---

\(^1\)Example borrowed from Bender & Orszag, *Advanced Mathematical Methods for Scientists and Engineers*. 
P2 (Zero state/zero input response and transfer function). Suppose $y(t)$ solves the constant-coefficient IVP
\[ ay'' + by' + cy = g(t), \quad y(0) = y_0, \ y'(0) = y'_0 \]
and let $G(s)$ be the Laplace transform of $g(t)$.

a) What is the characteristic polynomial $p(\lambda)$ for the ODE? 

b) Explain why $y$ can be written as the sum of two parts
\[ y = y_{ZSR} + y_{ZIR} \]
where $y_{ZSR}$ is the zero state response solving
\[ ay'' + by' + cy = g(t), \quad y(0) = y'(0) = 0 \]
and $y_{ZIR}$ is the zero input response solving
\[ ay'' + by' + cy = 0, \quad y(0) = y_0, \ y'(0) = y'_0. \]

Note: The terms here come from electrical circuits, where the ZSR is the response of the system to the input $g(t)$ from rest, and the ZIR is the response of the system to initial conditions without an input.

c) Show that the Laplace transform $Y_{ZSR}$ of the zero state response $y_{ZSR}$ is given by
\[ Y_{ZSR}(s) = H(s)G(s) \]
where $H(s) = 1/p(\lambda)$ (the transfer function).