SUPERRESOLUTION OF REMOTELY SENSED IMAGES WITH ANISOTROPIC FEATURES

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Abstract—We consider the problem of superresolution for remotely sensed images. Our ambition is to develop an algorithm that efficiently increases the resolution of an image without introducing artifacts or blurring, and without using any additional information, such as images of the same scene in different modalities or sub-pixel shifts of the same image at lower resolutions. The approach developed in this article is based on analysis of the directional features present in the image that is to be superresolved. The harmonic analytic technique of shearlets is employed in order to efficiently capture the directional information present in the image, which is then used to provide smooth, accurate images at higher resolutions. Our algorithm is compared to the standard superresolution method of bicubic interpolation. We test our algorithm on a remotely sensed image of Gulfport, Mississippi. Our results indicate the superior performance of shearlet-based superresolution algorithms, compared to bicubic interpolation.

Index Terms—Image processing, superresolution, shearlets, remote sensing, anisotropic dictionaries.

I. INTRODUCTION

This article develops a novel algorithm for the superresolution of images. A superresolution algorithm increases the resolution of an image, while attempting to preserve smoothness and important information in the image, and without introducing artifacts. There may be additional information incorporated into the new superresolved image, such as lower resolution sub-pixel shifts of the same scene, or images of different modalities. Many classical methods for superresolution employ an interpolation scheme, based on some form of weighted local averaging [1]. More sophisticated methods exploit the geometry inherent in the image to augment these interpolation schemes by improving smoothness [2].

In order to analyze the geometry of an image, anisotropic harmonic analysis techniques are useful. These methods provide directionally sensitive computational tools for decomposing an image, and efficiently and accurately encoding the most salient features. In particular, the construction of shearlets offers a computationally efficient method for analyzing the directional content of an image. This information can be used to provide smoother superresolved images, as our algorithm’s results demonstrate; see Section V.

The structure of this article is as follows. We begin with relevant background on the problem of image superresolution in Section II. We give a brief introduction to shearlets in Section III. Our shearlet-based superresolution algorithm is detailed in Section IV, and is tested on a remotely sensed image in Section V. We conclude and explore future work related to this algorithm in Section VI. All figures appear in Section VIII.

II. BACKGROUND ON SUPERRESOLUTION

The problem of superresolution is significant in image processing. The goal of superresolution is to increase the resolution of an image I, while preserving detail and without producing artifacts. The outcome of a superresolution algorithm is an image Ĩ, which is of the same scene as I, but at a higher resolution. We restrict ourselves here to greyscale images, hence we can consider our images as real-valued matrices. Let I be an M × N matrix and Ĩ an M × Ñ matrix, with M < M̃, N < Ñ. We consider the common case where M = 2M and N = 2N, which corresponds to doubling the resolution of the original image.

Superresolution can be done by using information in addition to I, such as low resolution images at sub-pixel shifts of the scene [3], or images of the scene with different modalities. The latter method is related to the specific problem of pan-sharpening [4]. Alternatively, superresolution can be performed using only I. The first type of superresolution requires additional data, and is thus less desirable than the second type. In this article, we shall develop a superresolution method of the second type, which requires as input only the image itself.

There are several standard approaches to superresolving I without using additional information. Among the most common are nearest neighbor interpolation and bicubic interpolation. Let us consider the superresolved version of I = {a_{m,n}}, denoted Ĩ = {ã_{i,j}}. Here, the values ã_{i,j} and a_{m,n} may be understood as entries of a real matrix representing the images. We must compute each pixel value in the new image, namely ã_{i,j}, from the pixel values of the original image, a_{m,n}.

In the case of nearest neighbor interpolation, new pixel
values are computed simply by replicating current pixel values. This method is simple and computationally efficient, but leads to extremely jagged superresolved images. It is unsuitable when a high-quality, smooth \( \hat{I} \) is required. Other methods involve convolving the image with an interpolation kernel, which amounts to taking a weighted average of pixel values within some neighborhood. For example, bicubic interpolation determines \( \hat{I} \) by computing each \( \hat{a}_{i,j} \) as a weighted average of the 16 nearest neighbors in \( I \); the weights are chosen to approximate the derivative values at the pixels being analyzed. A precise description of the algorithm may be found in [1].

A novel method for improving the smoothness of images superresolved using these methods was demonstrated in [2]. In this algorithm, local dominant directions are computed using tight frames derived from circulant matrices. After using nearest neighbors or bicubic upsampling, a motion blur filter is applied in the dominant direction. Areas with low variance are assumed to have no dominant direction. This method resulted in superresolved images with much smoother edge features when compared to the interpolation techniques alone. This method proved effective, but required new tight frames to be computed for each application of the algorithm, depending on the image size and the structure of the features present, thus limiting its efficacy. Essentially, this method uses frame theory to compute locally dominant directions; we shall compute locally dominant directions in another, more efficient manner.

The aim of this paper is to develop a superresolution algorithm that computes dominant directions efficiently and accurately, using the harmonic analytic construction of shearlets [5], [6], [7], [8]. This method is quite general, and can be applied to images of any size, and has few tunable parameters.

III. BACKGROUND ON SHEARLETS

Shearlets are a generalization of wavelets that incorporates a notion of directionality. We thus begin our mathematical discussion of shearlets with some background on wavelets [9].

In a broad sense, wavelet algorithms decompose an image with respect to scale and translation. Mathematically, for a signal \( f \in L^2([0, 1]^2) \), understood as an ideal image signal, and an appropriately chosen wavelet function \( \psi \), \( f \) may be written as

\[
f = \sum_{m \in \mathbb{Z}} \sum_{n \in \mathbb{Z}^2} \langle f, \psi_{m,n} \rangle \psi_{m,n},
\]

where:

- \( \psi_{m,n}(x) := |\text{det} A|^{m/2} \psi(A^m x - n) \).
- \( A \in GL_2(\mathbb{R}) \).

A typical choice for \( A \) is the dyadic isotropic matrix \( 2I_2 \), where \( I_2 \) is the \( 2 \times 2 \) identity matrix. The set of wavelet coefficients \( \{ \langle f, \psi_{m,n} \rangle \}_{m \in \mathbb{Z}, n \in \mathbb{Z}^2} \) describes the behavior of \( f \), our image signal, at different scales (determined by \( m \)) and at different translations (determined by \( n \)). This infinite scheme is truncated to work with real, finite image signals [10].

Shearlets generalize wavelets by decomposing with respect not just to scale and translation, but also direction. Mathematically, given a signal \( f \in L^2([0, 1]^2) \) and an appropriate shearlet function \( \psi \), we may decompose \( f \) as

\[
f = \sum_{i \in \mathbb{Z}} \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}^2} \langle f, \psi_{i,j,k} \rangle \psi_{i,j,k},
\]

where:

- \( \psi_{i,j,k}(x) := 2^{j/2} \psi(B^i A^j x - k) \).
- \( A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \).

Note that \( A \) is no longer isotropic, hence it will allow our new analyzing functions to be more pronounced in a particular direction. The new matrix \( B \), a shearing matrix, lets us select the direction. The shearlet coefficients \( \{ \langle f, \psi_{i,j,k} \rangle \}_{i,j,k \in \mathbb{Z}^2} \) describe the behavior of \( f \) at different scales (determined by \( i \)), translations (determined by \( j \)) and directions (determined by \( k \)). The anisotropic character of shearlets has proven useful for a variety of problems in image processing, including image denoising [11], and image fusion [12]. The ambition of this article is to apply shearlets to the problem of image superresolution.

IV. DESCRIPTION OF SHEARLET SUPERRESOLUTION ALGORITHM

Our algorithm for shearlet-based superresolution is coded in MATLAB. It is described in Figure 1. Consider an \( M \times N \) image matrix \( I \); the superresolved image shall be denoted \( \hat{I} \) as above.

V. EXPERIMENTS AND RESULTS

We test our algorithm against standard bicubic interpolation by running both supersolution algorithms on a remotely sensed image. Our test image is from an aerial view of the University of Mississippi-Gulf Park near Gulfport, acquired with a CASI-1500 sensor with spectral range 375 – 1050 nm in 72 bands. We choose to analyze the 30th band, due to its relatively high contrast. The image has a spatial resolution of 1 m and consists of 325 × 337 pixels. We perform our algorithm on the full spatial image, though we extract a 125 × 125 subset for visualization purposes; this subset is shown in Figure 2. The resulting 250 × 250 image produced from a simple bicubic interpolation is shown in Figure 3.

We consider three shearlet-based superresolved images, based on three methods for determining the pixels with no dominant direction (see Step 4 in Figure 1). In the interest of space, we show only the results for methods a.) and c.). The superresolved images using our algorithm are shown in Figures 4 and 6.

Figures 5 and 7 illustrate the local directions chosen by our algorithm with methods a.) and c.) for determining pixels with no dominant direction. In these images, the colors vary from dark blue (corresponding to a dominant direction of 90°) to dark red (corresponding to a dominant direction of 258.75°). In the case of method a.), only a few pixels, indicated by the
1) Apply the Fast Finite Shearlet Transform \([13], [14]\) to \(I\). This produces shearlet coefficients up to \(\left\lfloor \frac{1}{2} \log_2(\max\{M,N\}) \right\rfloor\) scales. If we label the scales from coarsest to finest scale starting at \(j = 1\), we have \(2^{j+1}\) matrices of size \(M \times N\) at the \(j\)th scale, each corresponding to a different direction from \(90^\circ\) to \((90 + 180(1 - 1/2^{j+1}))^\circ\), equally spaced. Denote these directional matrices \(D_1, \ldots, D_{2^{j+1}}\). For the experimental images, we used the \(j = 3\) scale since it best captured the edges, giving us 16 directions. This parameter may be set differently depending on the size of the image under analysis.

2) Upsample by a factor of 2 each of \(D_1, \ldots, D_{2^{j+1}}\), using the upsampling method of bicubic interpolation to acquire \(\tilde{D}_1, \ldots, \tilde{D}_{2^{j+1}}\). These contain the directional information that will be used later.

3) Upsample by a factor of 2 the original image \(I\), using the upsampling method of bicubic interpolation to acquire \(\tilde{I}\). This upsampled \(\tilde{I}\) will be modified using the directional information present in \(D_1, \ldots, D_{2^{j+1}}\).

4) Assign each pixel in \(\tilde{I}\) a local direction based on which matrix contains the shearlet coefficient of largest magnitude, i.e., which entry in that location is maximal among \(D_1, \ldots, D_{2^{j+1}}\). Pixels which have no dominant direction are determined by one of the following three methods:

a) Pixels whose maximum coefficients were in the bottom 10% of all max coefficients were assigned no direction.

b) Apply a standard deviation filter of size 5 to \(\tilde{I}\) to acquire \(I_\sigma\). If a pixel in \(I_\sigma\) has value less than .05, this pixel is assigned no dominant direction. Intuitively, pixels with low \(I_\sigma\) value are locally constant, and should not be assigned a dominant direction. The parameter .05 can be tuned as needed.

c) Apply the Canny edge detector with default parameters to \(\tilde{I}\), then thicken the edges using the MATLAB function ‘imdilate’. Apply this mask to each of \(\tilde{D}_1, \ldots, \tilde{D}_{2^{j+1}}\) and proceed as in a.) This has the effect of picking no dominant direction if a pixel is far from an edge-like feature.

5) Apply a motion blur filter of length 5 in each of the \(2^{j+1}\) directions \(\tilde{I}\), to produce \(\tilde{I}_1, \ldots, \tilde{I}_{2^{j+1}}\).

6) Replace the pixel values of \(\tilde{I}\) by their corresponding blurred version based on the previously assigned local direction, i.e. with the pixel value in \(\tilde{I}_m\), where the pixel has dominant direction corresponding to \(m\).

7) Output the superresolved image \(\tilde{I}\).

Fig. 1. Description of Shearlet Superresolution Algorithm.

darkest blue, were not assigned a direction. Some of these pixels can be seen in the lower left corner of the figure. In the case of method c.), far more pixels were not assigned a direction. Method c.) seems to be more accurate in finding all edges, when compared to method b.), which is not pictured in this article.

VI. CONCLUSIONS AND FUTURE WORK

Our shearlet algorithm produces smoother superresolved images with fewer artifacts, when compared to bicubic interpolation. Notably, the method using c.) in Step 4 in Figure 1, namely using a Canny edge detector to find areas with no dominant direction, provided very good results. In particular, the dominant direction map seen in Figure 7 is quite convincing in this case. We would like to find a quantitative measure that demonstrates the superiority of our algorithm over bicubic interpolation, which is clear visually. Towards this end, we cut out a \(125 \times 100\) subset out of the top left of each superresolved image, an area consisting mainly of edges and flat regions. See Figure 8 for this region in the superresolved image using bicubic interpolation. We then averaged the length of the gradient vector over all pixels using central differences. The idea is that jagged edges lead to longer edges and hence larger gradients. Smaller gradient vectors are then associated with smoother, more accurate edges.

Under this metric, the bicubic upsampling did the worst with an average gradient length of 0.0244. Methods b.) and c.) performed better, averaging 0.0218 and 0.0213, respectively. Method a.) performed the best with an an average gradient of 0.0199. Recall, however, that we restricted ourselves to an area with primarily edges and flat regions. For more complicated areas, method a.) tends to blur excessively, so we conclude that method c.) is the best overall. The images used for this quantitative analysis appear in Figures 8, 9, and 10. We also compared our results to the state-of-the-art Sparse Mixing Estimators method (SME) in [15]. We found that our method compared well in terms of smoothing the jagged edges, though SME produced an overall sharper image at additional computational cost. We believe that this is due to the directional interpolation employed by SME.

Generalizing this approach by using other anisotropic representation systems beyond shearlets, such as curvelets [16] and composite wavelets [17], is of interest. In addition, finding more sophisticated ways to implement the local directional information, beyond motion blurring, has the potential to improve superresolution results. This could be done through a variety of cutting-edge interpolation techniques [15], [18].

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REFERENCES


VIII. FIGURES

Fig. 2. Original 125 × 125 image of Gulfport, MS. This is I in the algorithm as described in Figure 1.

Fig. 3. Superresolved 250 × 250 image with bicubic interpolation. Notice that the edge-like features are jagged.

Fig. 4. Superresolved 250 × 250 image using our algorithm with a.) in Step 4.
Fig. 5. Assignment of local directions, based on maximal shearlet coefficients and a.) in Step 4. Direction varies from dark blue (90°) to dark red (258.75°). The darkest blue corresponds to no assigned direction.

Fig. 6. Superresolved 250 × 250 image using our algorithm with c.) in Step 4.

Fig. 7. Assignment of local directions, based on maximal shearlet coefficients and c.) in Step 4. In this case, a Canny edge detector is applied to determine which pixels have no dominant direction. Direction varies from dark blue (90°) to dark red (258.75°). The darkest blue corresponds to no assigned direction.

Fig. 8. The upper left 125 × 100 pixels of the superresolved image using bicubic interpolation. Average gradient is 0.0244.

Fig. 9. The upper left 125 × 100 pixels of the superresolved image using our algorithm with a.) in Step 4. Average gradient is 0.199.

Fig. 10. The upper left 125 × 100 pixels of the superresolved image using our algorithm with c.) in Step 4. Average gradient is 0.0213.