1. Determine when the integral \( \int_0^\infty e^{ct} \, dt \) converges. (Here \( c \) is a real constant.)

Definition: An improper integral is defined as

\[
\int_a^\infty f(t) \, dt = \lim_{A \to \infty} \int_a^A f(t) \, dt .
\]

If the limit exists, the integral is said to converge. Otherwise, it is said to diverge.

2. Discuss the integral \( \int_1^\infty t^{-p} \, dt \).

Definition: \( f \) is piecewise continuous on an interval if: (a) \( f \) has only a finite number of discontinuities; (b) At each discontinuity, \( f \) approaches finite limits from both sides.

Theorem 1: Suppose \( f \) is piecewise continuous for \( t \geq a \).
(a) If \( |f(t)| \leq |g(t)| \) and if \( \int_a^\infty g(t) \, dt \) converges, then \( \int_a^\infty f(t) \, dt \) also converges.
(b) If \( f(t) \geq g(t) \geq 0 \) and if \( \int_a^\infty g(t) \, dt \) diverges, then \( \int_a^\infty f(t) \, dt \) also diverges.
Definition: The Laplace transform of $f$ is given by

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t) \, dt . \quad (2)$$

3. Find $L\{f(t)\}$ for the following cases:
   (a) $f(t) = a$.
   (b) $f(t) = e^{at}$.
   (c) $f(t) = \cos at$.

Theorem 2: The Laplace transform $L\{f(t)\} = F(s)$ exists for $s > a$ provided that
   (a) $f$ is piecewise continuous on $0 \leq t \leq A$ for any $A > 0$;
   (b) $|f(t)| \leq Ke^{at}$ when $t \geq M$.

4. Find $L\{f(t)\}$ for $f(t) = 5e^{-2t} + 2\cos 5t$.
Comment: $L\{c_1f_1(t) + c_2f_2(t)\} = c_1L\{f_1(t)\} + c_2L\{f_2(t)\}$. That means, the Laplace transform is a linear operator.